MA110 – Mathematical Explorations – Spring 2014
Solutions and Proofs – Instructions for Writing and Details of Assessment

As described in the course syllabus, the focus of most of your work inside and outside of class is the investigations of mathematical problems. Some of these investigations will be large and open-ended problems, requiring significant attention over long periods of time to reach solutions. Other investigations will be smaller, more self-contained problems that you move through more steadily as you re-discover an important piece of mathematics.

A major component of your grade in this course comes from my assessment of your written solutions and proofs based on your mathematical work. This document explains why we write in mathematics class, what my expectations are from your written work, and how I will assess your work.

Reasons for Writing in Mathematics Class

> An elegantly executed proof is a poem in all but the form in which it is written.
> ~ Morris Kline

This is a core course - it is perfectly natural to expect students to write in core courses.

I am of the opinion that many people struggle with writing in the small – they have trouble concisely expressing their ideas at the level of individual sentences. This negatively impacts the quality of their writing in the large. Mathematics writing requires a high level of precision. I find that it offers a wonderful environment to work on exact writing and the precise expression of ideas.

In most professional settings you will have to write precise, intelligible reports. You will practice this here – your reports will focus on the progress of your mathematical discoveries.

Writing mathematics helps you learn mathematics. Communicating what you have learned helps you make sense of it.[1] Indeed, Fields medalist William P. Thurston declares, "human language is the number one factor important for mathematical thinking."[2]

Your Notebook

ALL of your mathematical work should be recorded in your notebook. The investigations require significant mathematical work. You will need a record of this work – your successes as well as your failures. Isaac Newton, Leonardo da Vinci, Rene Descartes, Srinivas Ramanujan – and many of the other great thinkers – carried notebooks where they recorded many of their thoughts. Mathematicians are still making

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[1] There is a wealth of literature on this. If you're interested in learning more, you may start with Writing to Learn Mathematics and Science by Paul H. Connolly and Theresa Vilardi, Teachers College Press, 1989.

[2] From "On Proof and Progress in Mathematics," *Bulletin of the American Mathematical Society*, 30:2 (1994), pp. 161-77.

breakthroughs in number theory by reading Ramanujan's notebooks[3], many of Newton's notebooks have just been made available online[4], Descartes notebook was "secret"[5],…

Your notebook should store all of your discoveries in this course.

It should be filled with patterns, data, tables, ideas, conjectures, connections, computations, proof strategies, and all of the other by-products of your work on the problems and investigations that are the foci of this course. You should record outlines and/or rough drafts of solutions and proofs as you go. Other than simple mistakes (e.g. arithmetic mistakes) you should not erase.

Your notebook will never be collected and graded. You should, however, keep your notebook well organized. This will help not only with your mathematical work, but with your subsequent written work too.

Generating Your Final Written Work

What is recorded in your notebook should be thought of as a rough draft of the work that you hand in to be graded. Generating the final drafts of your solutions/proofs will take significant time, just as meaningful writing in an area does.

Because it can be edited and revised upon proof-reading, I would suggest typing most of your final written work. Beware however, I expect much of your writing to include tables, images, diagrams, calculations and formulae. These are often difficult to generate, size, and import into basic word processors efficiently. I do not want you spending lots of time trying to do these things electronically. I.e. I do not expect you to use Equation Editor, Paint, Excel, etc. Instead, I encourage you to approach these things practically – feel free to handwrite things on your printed document and import pictures/images/diagrams into your document (perhaps from cell phone pictures you have taken).

Context for Your Solutions and Proofs

Here are several different settings that should help provide contexts within which you may view your writing efforts.

You *are* writing a report for someone else that completely documents your mathematical work. Reports of this type are written in many different professional settings - scientific lab reports, business sales reports, legal reports, etc. Your report must document and justify all of the significant findings of your work. Other than where it is supported by external references, the report should be entirely self-contained.

You may think of writing a narrative story which describes your mathematical work. Narratives have plots, characters, and themes that are described for the audience – so do your mathematical investigations. As a more precise example, you can think of the dramatic story arc where there is rising action (your initial work, as represented by what

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[3] See the chapter "Partitions" in Discovering the Art of Mathematics – Number Theory for examples.

[4] http://cudl.lib.cam.ac.uk/collections/newton

[5] Descartes Secret Notebook: A True Tale of Mathematics, Mysticism, and the Quest to Understand the Universe by Amir Aczel, Broadway, 2005.

is in your notebook), a climax (often a conjecture or observation), falling action (you working out the details of your mathematical work), and finally the resolution or denouement (which is often the proof in mathematics).

Certainly in the majority of your classes you must write expository and persuasive papers. In these papers you need to clearly state and then support/defend a thesis statement. Your proposed mathematical solution or conjecture is very much a mathematical thesis that your mathematical writing must precisely articulate and then adequately justify.

A final example is the "5W's" from journalism – Who, What, When, Where, Why and How. Who are the mathematical objects you are working with (whole numbers, triangles, tangent lines, symmetries, functions,…)? When and Where are these mathematical objects interacting/relating? This is an essential context that must be described for the reader. What is your fundamental observation, conclusion, conjecture or idea? Why is the What that you have descried happening? And How do you know this is Why it is happening – what is your justification or proof? All of these things should be part of your mathematics solutions and proofs.

Requirements

The following are requirements for your solutions/proofs. Thinking of this as a checklist may be useful in the early stages of your mathematical writing.

- The essential aspects of the problem/investigation are (re)-stated in the beginning of the explanation so your solution/proof can be read independently.
- Each sentence you write is true and you either explain why it is true or you provide an explicity reference to earlier work that you/we have completed.
- If you are not sure if a sentence is true, you have to state that, e.g. "I believe that…" or "I conjecture that…". Be aware though that your explanation can only be complete if you are certain that all your sentences are true.
- It is often helpful to include an example of your thinking to make the solution easier to understand for the reader.
- It is often helpful to include a diagram or picture to make the solution easier to understand for the reader. Draw neatly and describe the diagram/picture.
- Please show that you invested effort. If you cannot solve the problem you can for instance show all your attempts and explain what did not work.
- If your work is hand-written it must be neat and legible.
- Your explanation shows clear organization: In which order should the reader read the sentences and look at the pictures?
- The explanation is free of spelling and grammatical errors.
- All sentences are complete (not fragments), even if you write equations and refer to pictures.
- All quantities are clearly identified; in particular, the identity of all pronouns is unambiguous. For example: "*I know this works because it is going up*". What do you mean by "this"? And what do you mean by "it"?

- ∞ The explanation ends with a sentence that wraps it all up, e.g. "I think that I solved the problem correctly because…", "This answer makes sense to me, because…"

Grading Rubric

Your written work will be graded in the following five categories:

Mathematical Correctness and Completeness	Is your solution/proof mathematically correct? Are all of your sentences/claims legitimate? Is your solution/proof complete or does it contain gaps and/or limitations?
Depth of Understanding	Does your solutions demonstrate an understanding of both the problems at hand and your proposed solution/proof? Are there important aspects of the problem that you have neglected to consider?
Justification and Explanation	Have you justified your reasoning? Have you clearly explained your thinking? Does your writing compel the reader to believe that you truly understand the problem and have an appropriate solution/proof?
Coherence and Clarity	Are your solutions coherent and readable? Has your written work clearly expressed the mathematical intent of your solution/proof? Is the identity of objects you refer to clear?
Neatness, Organization, Grammar, Spelling, and Effort	The presentation and mechanics of your solutions are important. It is also crucial that your solutions indicate that you have expended sufficient effort in solving the problems and presenting the solutions as described here.

Notice that within this grading scheme approximately 60% of your grade is determined by purely mathematical issues. The other 40% is determined by issues related to your presentation of your mathematical solutions. If you do not make a conscientious effort to express your solutions in the format described above, you can receive a failing grade even if your solutions are "correct" in a mathematical sense.

Marks and Abbreviations on Your Graded Paper

I will often use shorthand, including:

? or So? = I don't understand what you are saying or your point is not clear.

Red. = Redundant; you have already essentially said this.

Sp = Circled word is not spelled correctly.

¶ = Need a new paragraph starting here.

When you have solutions for a large number of shorter investigations I will grade only a selected sampling of your solutions. In this case each solution is typically worth 3 points. I will leave the following marks that indicates how much credit you have received adjacent to each of the solutions I have graded:

+ = Practically perfect in every way; full credit.

✓ = Good, but with at least one significant defects; -1 point.

− = Multiple, significant defects/errors; -2 points.

✗ = Completely incorrect, unintelligible and/or deficient; -3 points.

Points will be taken from the appropriate categories in the rubric above so it will be clear to you from the cumulative scores in which areas you may need to improve.

Sample Solutions

The following are examples of appropriate written solutions to selected investigations from the Chapter 1: Fibonacci Numbers in Discovering the Art of Mathematics – Number Theory.

9. I think it would be tedious to determine the fiftieth Fibonacci number because, by definition, it is the sum of the previous two Fibonacci numbers. And, in this same way, I only know how to find these earlier Fibonacci numbers from finding all of the Fibonacci numbers before them. So it seems I would have to find all of the first 49 Fibonacci numbers to be able to determine the fiftieth.

25. By direct computation, the sum of the first four Fibonacci numbers is $1 + 1 + 2 + 3 = 7$.

Classroom Generated Sample Solutions

In the space below, record some of the sample solutions that you generated and reviewed with peers in class.