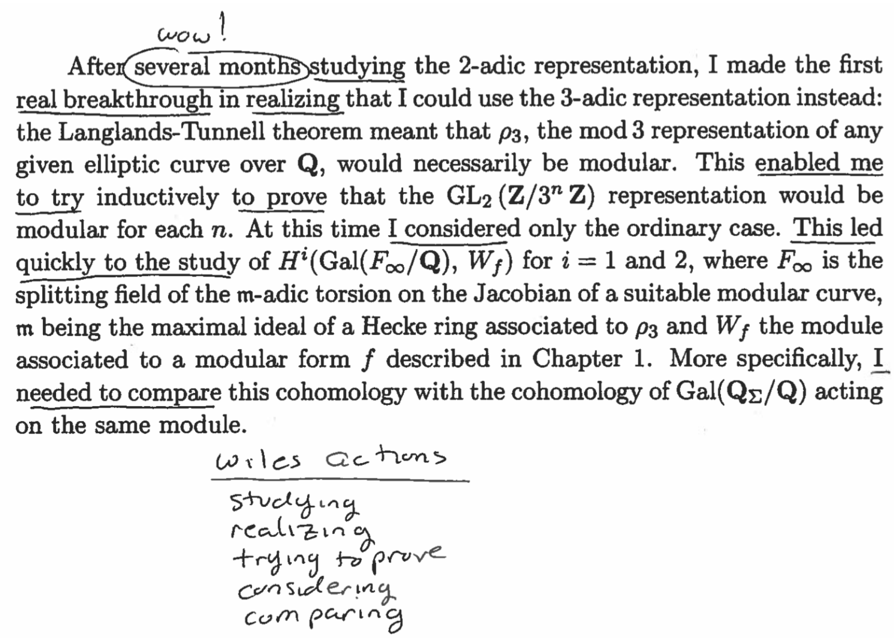
MA110
Mathematical Explorations
Spring 2016
Fermat's Last Theorem

1. Read pages 449 - 454 of Wiles' introduction to his groundbreaking paper ``Modular Elliptic Curves and Fermat's Last Theorem'' which appeared in the prestigious *Annals of Mathematics*, Vol. 142, pp. 443-551, 1995. As you read, annotate this introduction **focusing on the process of doing mathematics**. The majority of the nouns will be unfamiliar to you, and that is ok. It is not important that you have any idea what a ``2-adic [Galois] representation'' means. Moreover, the structure of the actual proof will not be clear to you; indeed, a year after Wiles' work was announced there was an intensive six-week workshop for Ph.D. mathematicians who wanted to understand his work. Rather, it is important for you to focus on the **process described**. You should highlight all of the words that you believe are related to learning, knowledge, discovery, and understanding. An example is shown below.


wow!

After several months studying the 2-adic representation, I made the first real breakthrough in realizing that I could use the 3-adic representation instead: the Langlands-Tunnell theorem meant that ρ_3, the mod 3 representation of any given elliptic curve over $\mathbf{Q}$, would necessarily be modular. This enabled me to try inductively to prove that the $GL_2(\mathbf{Z}/3^n\mathbf{Z})$ representation would be modular for each n. At this time I considered only the ordinary case. This led quickly to the study of $H^i(\mathrm{Gal}(F_\infty/\mathbf{Q}), W_f)$ for $i = 1$ and 2, where F_∞ is the splitting field of the $\mathfrak{m}$-adic torsion on the Jacobian of a suitable modular curve, $\mathfrak{m}$ being the maximal ideal of a Hecke ring associated to ρ_3 and W_f the module associated to a modular form f described in Chapter 1. More specifically, I needed to compare this cohomology with the cohomology of $\mathrm{Gal}(\mathbf{Q}_\Sigma/\mathbf{Q})$ acting on the same module.

Wiles actions

studying
realizing
trying to prove
considering
comparing

2. For **Thursday, February 18**, write a 2 – 3 page paper that compares and contrasts your experiences doing mathematics in your previous mathematics courses with those of Andrew Wiles (as described in his paper and in the video ``The Proof''). Your piece should have a coherent theme that is supported by evidence from your experiences, the readings and the video.

This assignment is worth 15 points and will be graded using the following rubric:

- Clarity of theme and coherence of argument – 5 points.
- Depth and richness of the comparison/contrasts – 5 points.
- Effort, grammar, spelling and organization of your piece – 5 points.