25/25

Maypole Dance Proof #2

What does math have to do with the maypole dance? I have spent the past month, alongside some of my peers and professors answering this exact question. I have finally come to some conclusions and conjectures that I believe are not only true but fascinating.

If you would like to know some of the intial work that took place please refer to maypole Proof #1

One of our intial challenges was developing a way to represent or draw a dance on paper, so that we could test conjectures and ask questions without having to do the dance everytime.

Figure 1 shows how our class chose to represent any maypole dance. This can represent all maypole patterns because...

1) This is not color based. although it is tempting to color these diagrams in (and often very helpful) this diagram is independent of any variables, like color, that could confuse results.

2) because #'s represent leaders and letters represent followers in all cases.

not dummy ribbon swap?

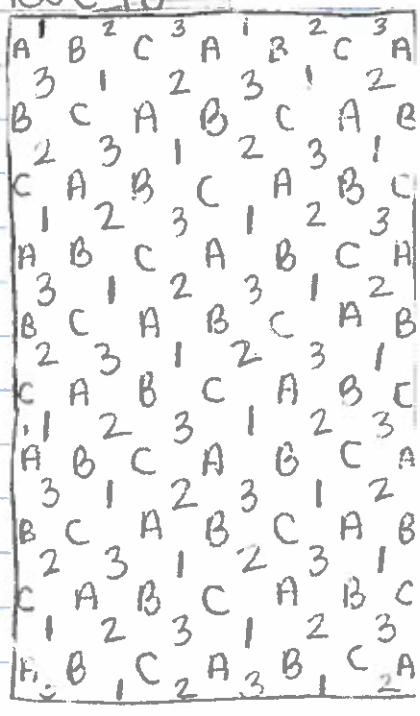


FIG 1A

The term leaders refers to ribbons which go over first in the dance. accordingly, followers go under first in the dance. Therefore:

A, B, C	and	1, 2, 3
are followers		are leaders

in the case of figure 1. From here, you can decide, from figure 1, the pairings.

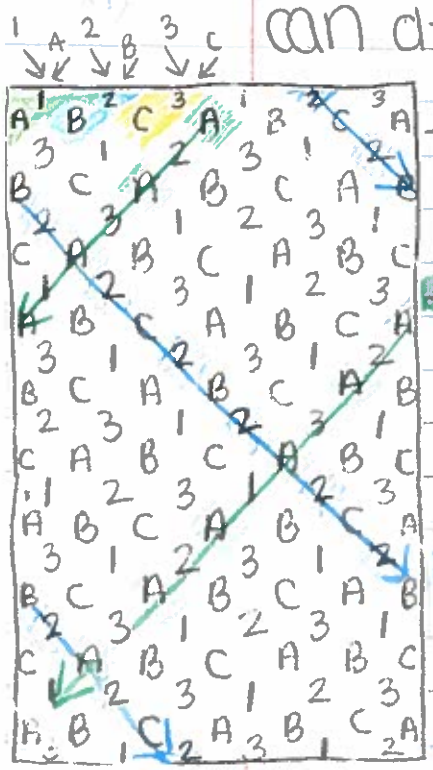


FIG 1B

- as you can see, the #'s pass over the letters on the first turn. Then the pattern repeats over-under
- all of the letter ribbons go ↙ down the maypole. If you were to follow the second A ribbon (note this diagram shows a repeated 1A 2B 3C pattern) you would see how it travels thru the pic
- all of the number ribbons go ↘ down the maypole. If you were to follow the second 2 ribbon, you would see how the ribbon travels through the picture.

We can also represent all of this through writing.

1A-2B-3C

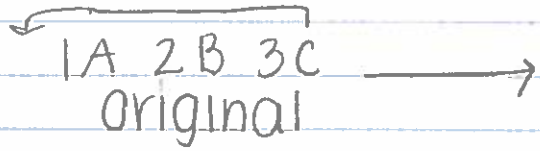
- tells us #'s (listed before each letter) are leaders.
- tells us the 1 goes over A as its first step and these two are named partners. (same with 2B and 3C)

- for 3 couples

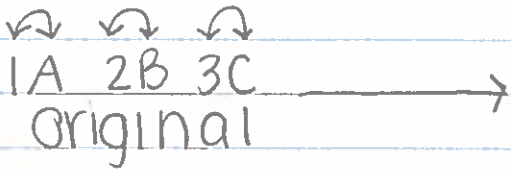
1A 2B 3C is the basic unit of dances. From this point we began to wonder, what would happen if we switched the order of ribbons or made letters leaders? Essentially, we wanted to mess with the 1A-2B-3C pairings and see what would happen or what kinds of patterns would emerge. and if the patterns would be the same or not.

As a class, we developed 3 main ways you can manipulate pairings, ribbon order, etc.

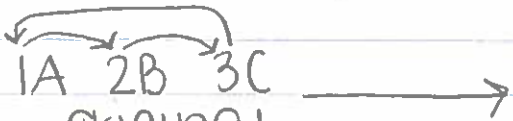
letter rotation



ribbon swap



leader rotation



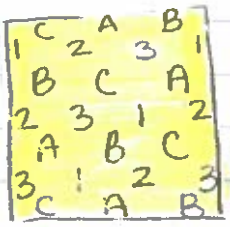
✳ PICTURES ✳

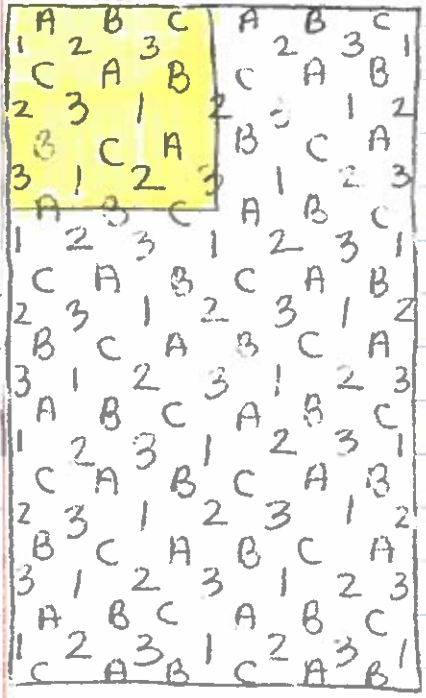
Fundamental Domains allow us to talk about these three variations in smaller units However this unit still completely represents the whole The fundamental domain is the repeating pattern in the whole

letter rotation

C1 A2 B3

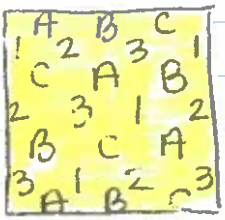
Fundamental Domain





ribbon swap

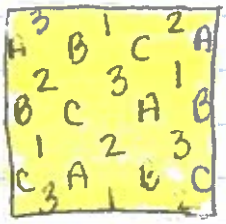
A1 B2 C3



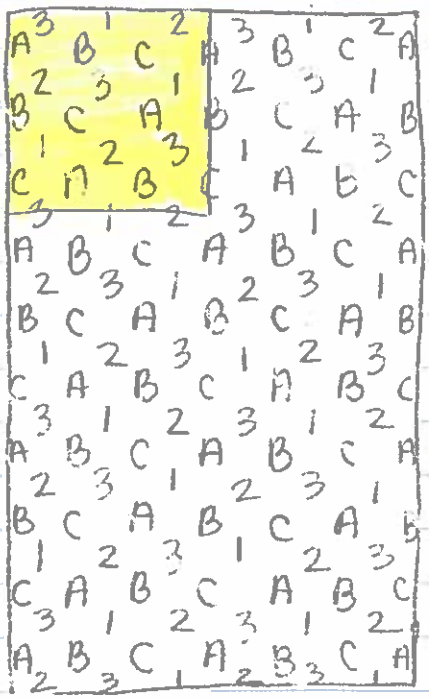
Fundamental Domain

leader rotation

3A 1B 2C



Fundamental Domain



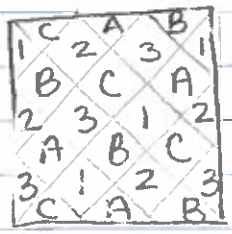
f.d. - fundamental domain

Now, I am going to show you the conjectures we developed regarding each manipulation of pairings.

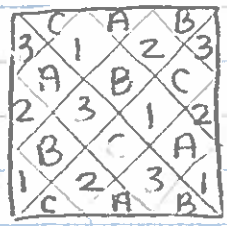
For each one, we tried to manipulate the letter rotation, ribbon swap, or reader rotation
why? what was our goal?

Letter Rotation C1 A2 B3

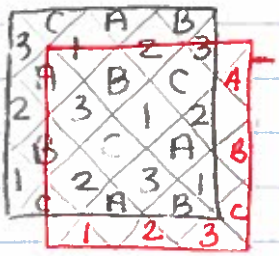
In order to get to the original f.d., we must...



take the F.D. of letter rotation



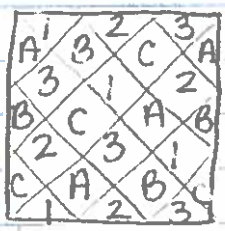
reflect it across the axis. (or 180°) ✓



shift F.D. up 1.

↑ direction

=

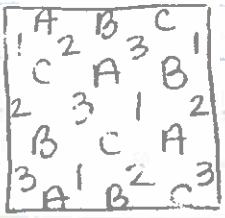


original F.D.

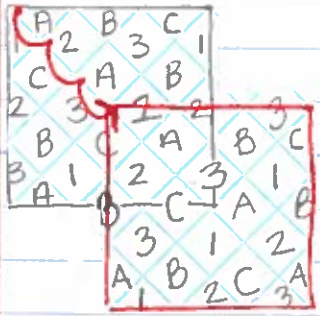
* This brings you to letter rotation. Follow steps accordingly to get to the original Fundamental Domain

Ribbon Swap | A1 B2 C1

In order to get to the original F.D. we must...

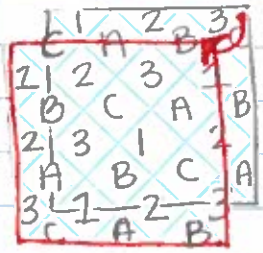


take the F.D of ribbon swap



shift F.D. down 3

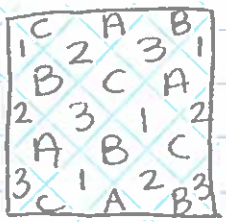
↘ direction



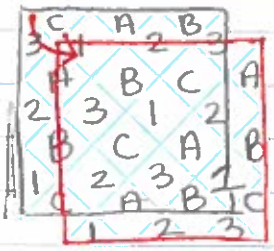
* then shift down 1

↙ direction

same as letter rotation



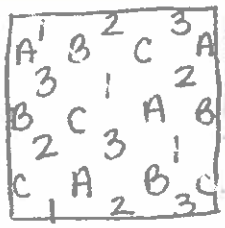
reflect this along axis or 180°



shift F.D. up 1

↖ direction

=

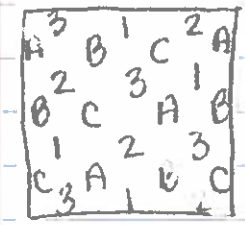


original F.D.

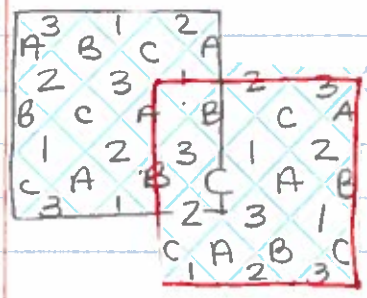
*

Leader Rotation 3A 1B 2C

In order to get to the original F.D. we must...



Take the Fundamental Domain

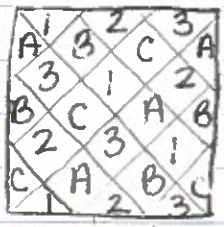


Shift F.D. up 4

↖

direction

=



original

Why is leader rotation just a shift whereas letter rotation and ribbon swap require a reflection, then shifts?

3A 1B 2C

↑ ↑ ↑

In this case, numbers remain leaders, therefore a simple shift will allow us to simply "hunt" for the original F.D in other words, leader rotation exists within Fig 1, the it just starts at a different pairing. or spot in dance. Letter rotation and ribbon swap on the other hand, have letters as leaders, therefore we need to reflect the picture to get the right #, letter pairing order.

Nice!

Conclusion

In conclusion, shifts and reflects are necessary to manipulate a ribbon swap, letter or leader rotation back to the original fundamental domain. Essentially, all these patterns (ribbon swap, etc) are found within the infinite fundamental domain of 1A2B3C, thru manipulation. I know this because otherwise it would be impossible to convert fundamental domains of the manipulated pairings to the original. Our process only involves a direct, 180° reflection or a shift diagonally up (↗↗) or down (↘↙). There is so switching of #s or letters or any kind of deletion.
It is also important to note that the ribbon swap process involves the letter rotation pattern. In order to get to the original fundamental domain from the ribbon swap, you would shift the picture down (↘) 3 and down 1 (↙). This manipulation would give you the pattern for letter rotation C1 A2 B3. From here, you follow the steps of letter rotation to get to the original fundamental domain.
Furthermore- letter rotation's fundamental domain can actually be found within ribbon swap.

nice!

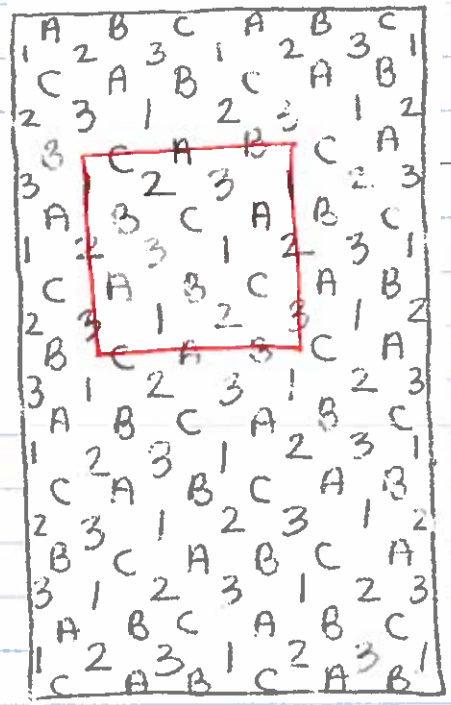
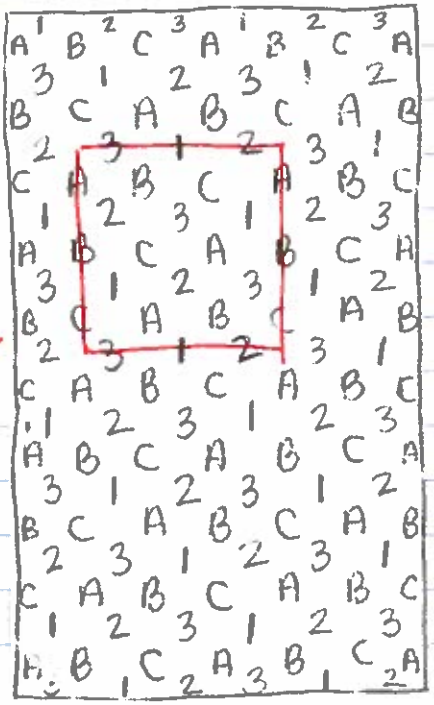


Fig 2

FIG 2 shows that letter rotation F.D C1 A2 B3 is found within ribbon swap

Lastly, leader rotation requires only a shift because the leaders (#'s) are still leaders. 3A 2B 1C. In this case, we are just looking at a later point in the dance where 3 meets with A, 2 with B and so on.