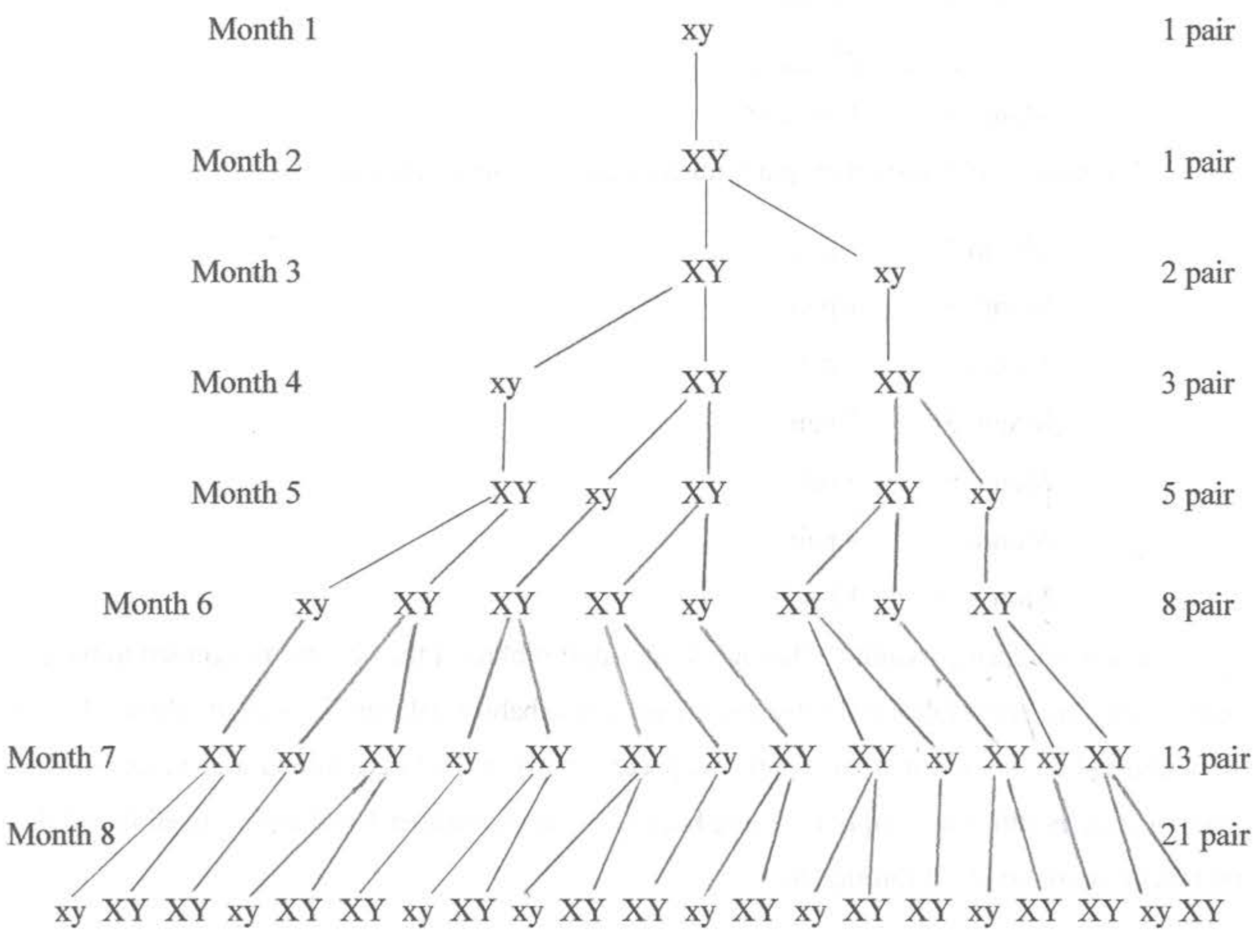
3
5
1
4
4

17/25

Chapter 1: Fibonacci Numbers

1.4 Fibonacci's Rabbits

1) The following tree diagram is a visual of how many rabbits were produced over the span of 8 months. Each pair of juvenile rabbits is represented by *xy* and each pair of mature rabbits are represented by *XY*.



2) The answer to Fibonacci's question: "how many pairs will be produced in a year?" is 144 pairs. I determined this answer by referring to Figure 1.2 and continuing the Fibonacci pattern as if I were continuing to draw out the tree diagram to 12 months. Since I know that at 8 months, there would be 21 pairs produced, I added the previous Fibonacci number (which was 13) to it to get month 9. As you can see below, this is how I determined that 144 pairs would be produced in a year.

Month 8	21 pairs
	+ 13
Month 9	34 pairs
	+ 21
Month 10	55 pairs
	+ 34
Month 11	89 pairs
	+ 55
Month 12	144 pairs

3) The number of adult rabbit pairs for months 2 – 8 are as follows:

Month 2:	1 pair
Month 3:	1 pair
Month 4:	2 pair
Month 5:	3 pair
Month 6:	5 pair
Month 7:	8 pair
Month 8:	13 pair

I noticed during months 2 through 8, the adult rabbits at the beginning decided to have a baby. Every time an adult has a baby, eventually those babies will turn into adult rabbits. This is what allows the number of adult rabbits to appear rapidly. How I determined the number of adult pairs in each month, was simply by going back to question number 1 and counting all the adult rabbits through months 2 through 8.

4) The number of adult rabbit pairs in a particular month can be determined by the previous month because I know that adult rabbits always continue onto the next month while also having a juvenile rabbit. Juvenile rabbits become mature in the following month after they are born. One juvenile rabbits become adult rabbits, the cycle continues. If you count the total number of mature adults and juvenile rabbits in a given month, then you will get a total of adult rabbits for the following month.

5) The number of juvenile rabbit pairs for months 2 – 8 are as follows:

Month 2:	0 pairs
Month 3:	1 pair
Month 4:	1 pair
Month 5:	2 pairs
Month 6:	3 pairs
Month 7:	5 pairs
Month 8:	8 pairs

I noticed through months 2 through 8, the juvenile rabbits have developed their own Fibonacci pattern just as the adult rabbit pairs did in question number 3. The only difference between the adult pairs and juvenile pairs is that the juvenile rabbit pairs are behind one month in the pattern; the juvenile pattern started with 0 while the adult pattern started with 1.This is because juvenile rabbits were not being produces in month 2, due to the adult rabbit just maturing.

6) The number of juvenile rabbit pairs in a given month can be determined by the number of rabbit pairs in earlier months by counting the number of mature adult rabbits in a particular month. By counting the ~~amount~~ number of adult rabbits, this will give you the number of juvenile rabbits for the next month because all mature rabbits produce a juvenile rabbit the following month. Unclear, +

7) The number of pairs of rabbits must also follow the defining relation $f_n = f_{n-1} + f_{n-2}$ of the Fibonacci numbers because this is the equation used to find them. The f_{n-1} of the formula is going to be used to find the number of adult rabbits in a ~~particular month~~ month n. You

Reference #7 and #6 where you have already explained this!!

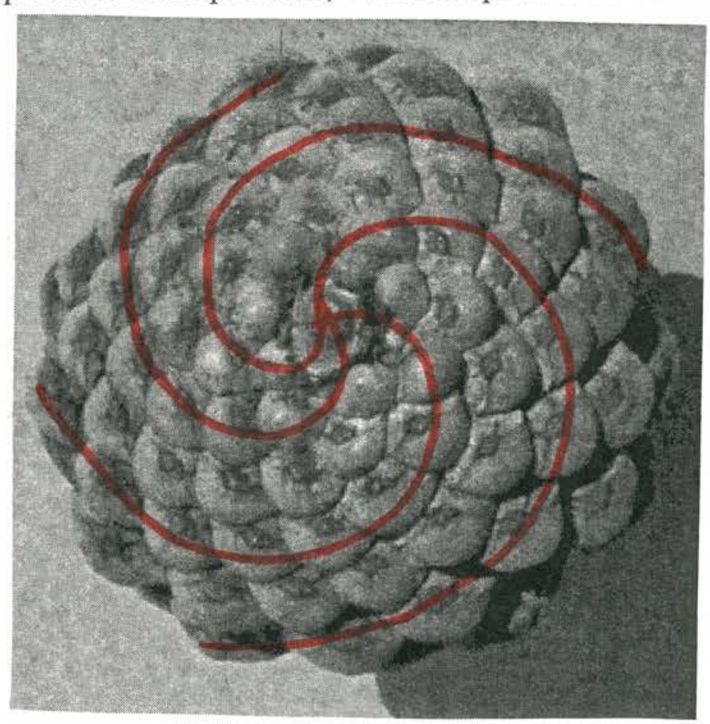
can determine that by going back one month, hence the n-1. The f_{n-2} of the formula is used to determine the amount of juvenile rabbits because you have to go back two months since it takes a month for juveniles to mature and have babies the following month. Once you add these two values together, then you will come up with the next Fibonacci number.

8) The twentieth Fibonacci number is 6,765 because I continued the Fibonacci pattern that was started with the breeding tree in investigation 1 until I got to the twentieth Fibonacci number; added the eighteenth and nineteenth Fibonacci number together. Say what these are.

9) I believe that it would be quite difficult to find the fiftieth Fibonacci number because in order to create a Fibonacci number sequence, you need to add two previous Fibonacci number together. This means, in order to find the fiftieth Fibonacci number, you would first need to find the 49 Fibonacci numbers before it. This would take a great deal of time and simple math if there was no formula or calculator involved.

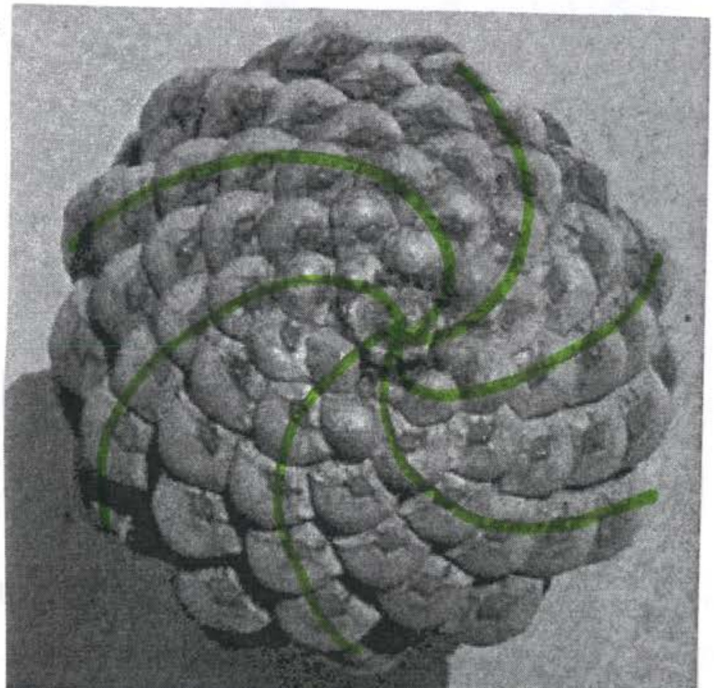
1.4.1 Fibonacci Spirals in Nature

10) Using a colored highlighters, I colored one of the spiral arcs in the pinecone that moves in a clockwise manner for the outer edge to the middle, while also skipping over the spiral that is adjacent to the one I just colored. I continued all the way around the pinecone until I had as many non-adjacent spiral arcs colored as I could. There are 8 clockwise spiral arcs in the pinecone; 4 colored spirals and 4 non-colored spirals.



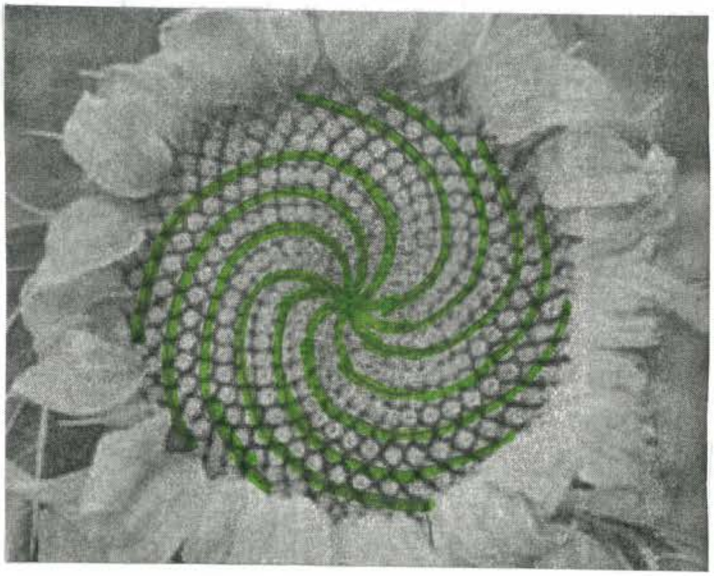
Backwards.

11) Using a different colored highlighter than from investigation 10, I took another image of a pinecone and colored the counter-clockwise spiral arcs just the same way as I did for the previous question. There are 13 counter-clockwise spiral arcs; 6 colored spirals and 7 non-colored.



12) Repeating the same directions as in investigation 10 for the sunflower, I found that there are 34 clockwise spiral arcs; 17 colored spirals and 17 non-colored spirals.

13) Repeating the same directions as in investigation 11 for the sunflower, I found that there are 21 counter-clockwise spiral arcs; 10 colored spirals and 11 non-colored spirals.

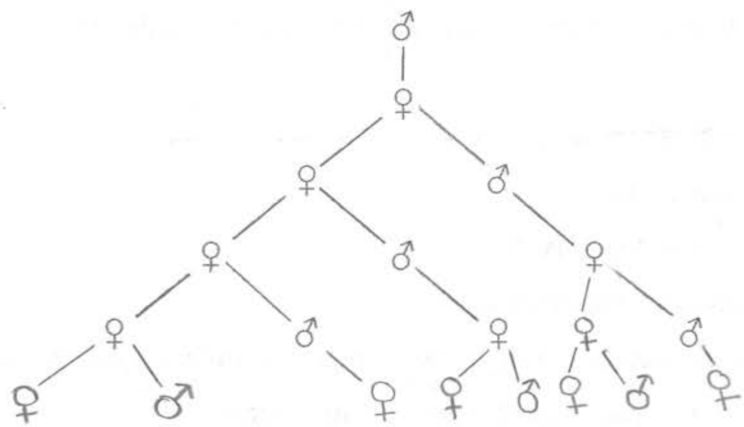


14) My answers for investigation 10 – 13 are surprising because when all the spirals (colored and non-colored) are added up, they equal a Fibonacci number. Also, comparing the pinecone to the sunflower, there are a greater amount of spirals on the sunflower that the pinecone. I believe this is because the seeds are smaller on the sunflower that the cone scales on the pinecone.

15) Yes, I would be surprised to learn that the number of spiral arcs in practically all pinecones and sunflowers are Fibonacci numbers. Some other examples are: honeycomb, rose pedals, aloe plants, conch shells, and pineapples.

1.4.2 Honeybee Family Tree

16) Using the standard symbols for male and female, ♂ and ♀, the following diagram is a family tree of a male bee that goes back five generations.



17) Using the family tree from investigation 16 determine the number of:

a. Parents: 1

b. Grandparents: 2

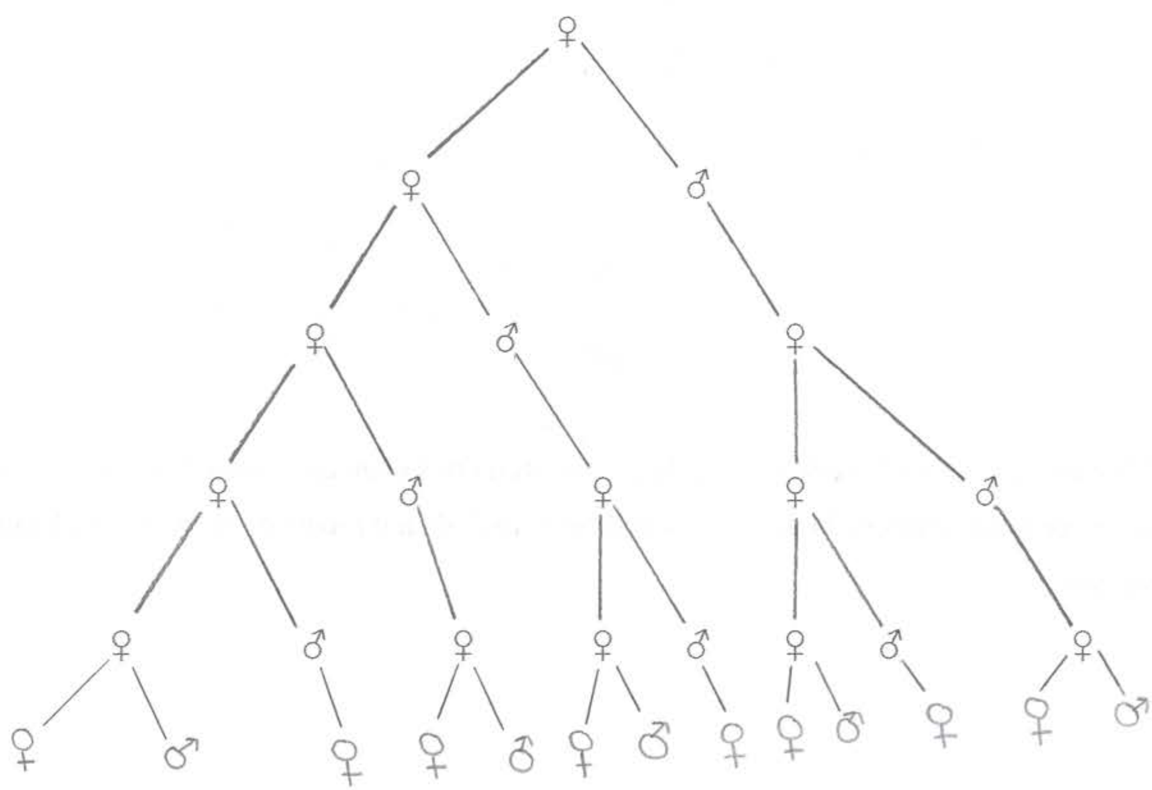
c. Great grandparents: 3

d. Great-great grandparents: 5

e. Great-great-great grandparents: 8

I notice that these numbers are Fibonacci numbers.

18) Using the standard symbols for male and female, ♂ and ♀, the following is a family tree diagram of a female bee that goes back five generations.



19) Using the family tree from investigation 18, determine the number of:

a. Parents: 2

b. Grandparents: 3

c. Great-grandparents: 5

d. Great-great grandparents: 8

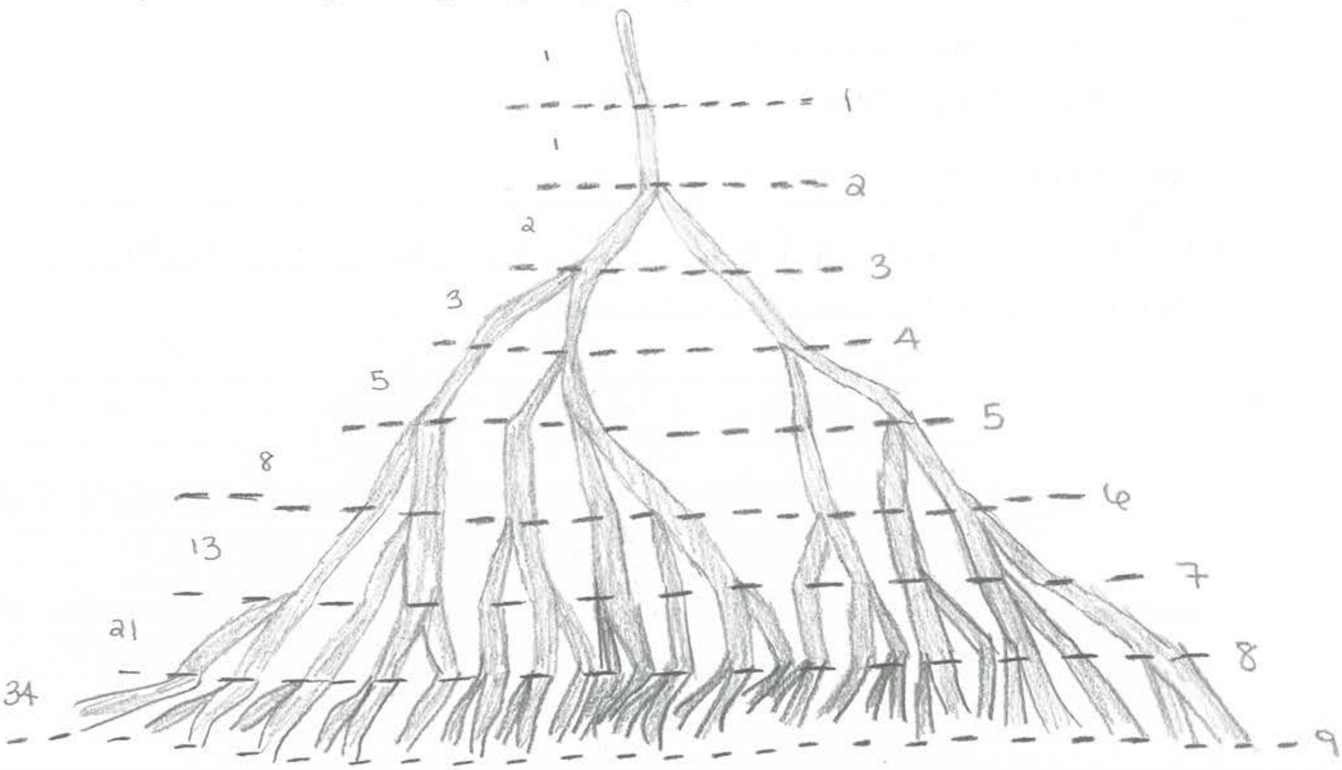
e. Great-great-great grandparents: 13

I notice that these numbers also follow the Fibonacci number sequence, but they just start and end at different Fibonacci numbers that in investigation 17.

than,

1.4.3 Plant Growth

20&21) The following drawing is a plant growing after 9 weeks.



22) At the end of each week, the number of shoots on this plant creates the Fibonacci number sequence because after a branch grows and "matures", then it produces another branch (after 2 weeks).

23) The number of shoots is always a Fibonacci number because a branch takes two weeks to divide, therefore once the branches divide it adds more to the sequence.

1.4.4 Two Fibonacci Identities

24) 1 + 1 + 2 = 4

25) 1 + 1 + 2 + 3 = 7

26) 1 + 1 + 2 + 3 + 5 = 12

27) 1 + 1 + 2 + 3 + 5 + 8 = 20

28) 1 + 1 + 2 + 3 + 5 + 8 + 13 = 33

29) The sums in Investigation 24 – 28 are related to Fibonacci numbers because you are just adding Fibonacci numbers to find the next Fibonacci number in the sequence.

Conjecture:

$$1 + 1 + 2 + 3 + \ldots + f_n = f_{n+2} - 1$$

30) 1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 = 54 = 33 + 21

Yes, they are both the same equation because whether you just have the number 33 or break it down into smaller numbers (1 + 1 + 2 +.....) and add 21 to both side, you still end up with the same sum of 54.

31) If you add the $(n + 1)$th Fibonacci number to both sides of the equation in investigation 29, this will generate the correct next stage. This is because when you add f_{n+2} and f_{n+1} it will equal f_{n+3} which just means the next Fibonacci number. Nice

+

$$1 + 1 + 2 + 3 + \ldots\ldots + f_n + f_{n+1} = f_{n+2} - 1 + f_{n+1}$$

what?

$$= f_{n+3} - 1,$$

32) This does prove that the results are correct for all values of n because this equation will always solve for the next Fibonacci number in the sequence. why?

which?

—

33) This diagram is showing the next three rows in Pascal's triangle.

				1					0th
			1		1				1st
		1		2		1			2nd
		1	3		3	1			3rd
	1	4		6		4	1		4th
	1	5	10		10	5	1		5th
1	6	15		20		15	6	1	6th
1	7	21	35		35	21	7	1	7th
1	8	28	56	70	56	28	8	1	8th

34) According to the 3rd row in Pascal's triangle, $(x+y)^3$ expanded would be:

$1x^3 + 3x + 3y + 1y^3$

35) A conjecture about the expansion of $(x+y)^9$ 6 is: WHY 256?

The expansion of $(x+y)^9$ 6 is going to be$1x^6 + 6x + 15y + 20xy + 15x + 6y + 1y^6$. —

36) When adding the individual rows of Pascal's triangle, the pattern that I see is that each row (starting after row 0) is multiplied by 2 of the previous row's total:

$0^{th} = 1$	$3^{rd} = 8$	$6^{th} = 64$
$1^{st} = 2$	$4^{th} = 16$	$7^{th} = 128$
$2^{nd} = 4$	$5^{th} = 32$	$8^{th} = 156$

+

37) The sums of the first and second shallow diagonals are 1 and 2.

38) The numbers that make up the third shallow diagonal are 1 and 2; their sum is 3.

39) The numbers that make up the fourth shallow diagonal are 1, 3 and 1; their sum is 5.

40) The numbers that make up the fifth shallow diagonal are 1, 4, and 3; their sum is 8.

41) I notice that the sums of the shallow diagonals form the Fibonacci number sequence. ✓

(As shown in #37 – #40)