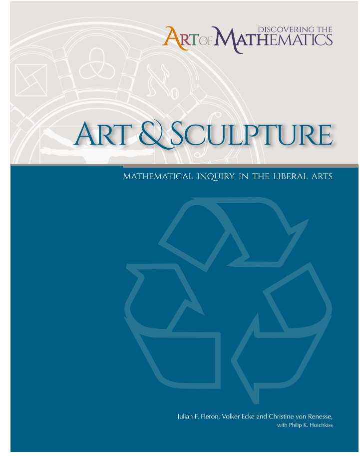
The following is excerpted from:

Discovering the Art of Mathematics: Art & Sculpture



By Julian F. Fleron, Volker Ecke, and Christine von Renesse, with Philip K. Hotchkiss.

As with all of our learning guides, this book is freely available online at

`http://www.artofmathematics.org/books/`

Discovering the Art of Mathematics (DAoM) is an NSF supported project that supports inquiry-based learning (IBL) approaches for mathematics for liberal arts (MLA) courses.

The DAoM curriculum consists of a library of 11 inquiry-based learning guides. Each volume is built around deep mathematical topics and provides materials which can be used as content for a semester-long, themed course. These materials replace the typical lecture dynamic by being built on inquiry-based investigations, tasks, experiments, constructions, data collection and discussions.

DAoM also provides a wealth of resources for mathematics faculty to help transform their courses. Extensive online resources include volume specific teacher notes and sample solutions, classroom videos of IBL in action, sample student work, regular blogs about teaching using IBL and a regular newsletter. Opportunities for supported reviewing and beta testing are also available.

For departments interested in IBL, DAoM offers traveling professional development workshops.

Full information about the *Discovering the Art of Mathematics* project is available at

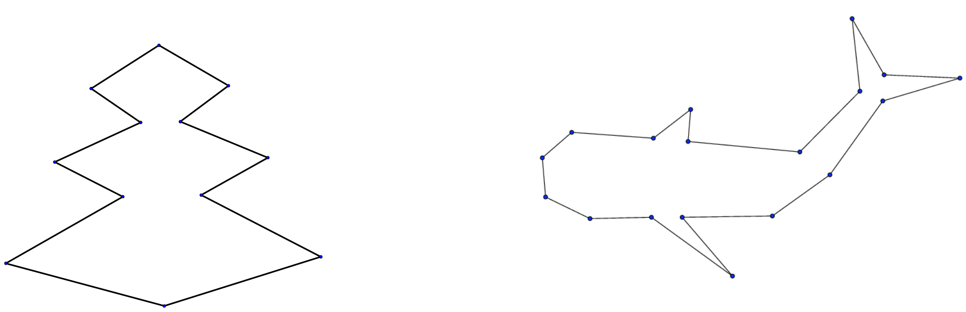
`http://www.artofmathematics.org`

This work is supported by the National Science Foundation under awards NSF0836943 and NSF122915. Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the author(s) and do not necessarily reflect the views of the National Science Foundation. These materials are also based on work supported by Project PRIME which was made possible by a generous gift from Mr. Harry Lucas. © 2009–2014

Use the excerpt below to explore for yourself how our materials can engage students in mathematical inquiry.

Origami: The Fold-and-cut Problem

In this chapter, we will investigate the **Fold-and-cut Problem**, sometimes also called the problem of Straight-Cut Origami*: Given a shape, such as those in the figure below, is there a a way to fold it up so that you can cut it out with a single straight cut? If there is such a way, how do we actually fold it? How can we describe all the shapes for which this is possible?



(a) Japanese *sangabisi* crest. (b) Whale for Straight-cut Origami.

*We learned about this problem from a 2005 New York Times article about the mathematician Eric Demaine.

If you boldly tried to fold the whale shape in order to cut it out with a single straight cut, you may have realized that this is not so easy. Often in mathematics, when we encounter a problem that seems too difficult to tackle with the tools we have at hand, we shift to a somewhat easier problem, in the hope of gaining a deeper understanding of the problem and of learning new tools. Let us therefore start with some shapes that are less complex and that exhibit some symmetry.

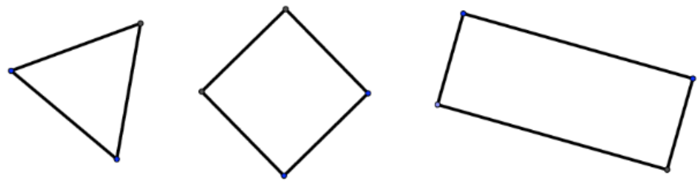


Figure 1. Three Geometric Shapes for Straight-cut Origami.

Figure 1 shows three simple geometric shapes: an equilateral triangle, a square, and a rectangle.

1. Find a way to fold up the ***equilateral triangle*** (three sides of the same length) in Figure 1 so that you can cut it out with a single straight cut. Once you succeed, clearly mark on both pieces of paper (the triangle and the outside) the actual fold lines you used. Explain your strategy.

2. As before, find the straight-cut folding for the square. Explain your strategy.

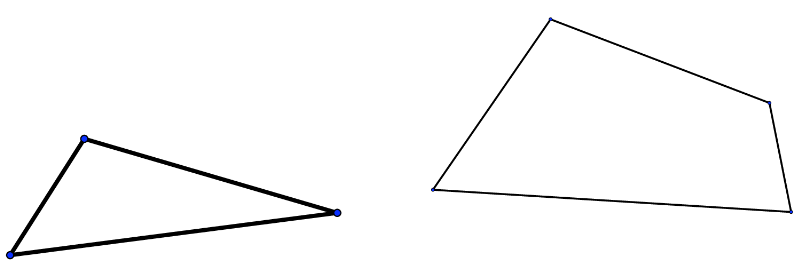
3. Now, find the straight-cut folding for the rectangle. Explain your strategy.

4. Take a new printout of each shape, draw in all the lines of reflection. What do you notice when you compare these with your fold lines? Write down everything you notice.

5. Classroom Discussion: Share your observations about straight-cut folds for symmetric shapes. Do you feel that you would be able to fold and cut any polygon with at least one line of symmetry? Try a few examples to check whether your conjecture is correct.

We noticed that lines of symmetry are very helpful in folding polygons so that we can cut them out with a single straight cut. Yet, with shapes that are less symmetric, it is no longer clear how to proceed.

You may find the following series of investigations more challenging than previous ones. It may take you more than just one attempt with each shape, sometimes *many* more. You may observe that using ideas gained from working with symmetric shapes may not be enough to fold these irregular shapes. Do not be discouraged. Where could you go for some new ideas? Do not discard the results of your attempts. Instead, use them as resources to analyze carefully what the results look like, and why they do not *completely* accomplish the task. Also keep an eye on what happens to interior and exterior areas when folding.



(a) Fold this irregular triangle. (b) Fold this irregular quadrilateral.

6. Find a way to fold the ***irregular triangle*** in the figure above (three sides all of different lengths).

7. Classroom Discussion: Other than angle bisectors, what other kinds of lines did you use to fold up the irregular triangle? How would you describe those?

8. An angle bisector is a line of symmetry for one angle. Is it also a line of symmetry of the *entire* shape? What are the consequences of folding along an angle bisector all the way through the shape?

9. Find a way to fold the ***irregular quadrilateral*** in the above figure (four sides all of different lengths).