Discovering the Art of Mathematics

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http://artofmathematics.wsc.ma.edu/

Funded by a National Science Foundation curriculum development grant, the *"Discovering the Art of Mathematics"* project[1] is developing a library of ten inquiry-based learning guides whose primary use is intended to be with collegiate Mathematics for Liberal Arts students. The following six learning guides are now ready for beta-testing:

Number Theory,	The Infinite,	Knot Theory,
Games and Puzzles,	Music and Dance,	and Geometry.

PDF copies are available for review from our web site at
http://artofmathematics.wsc.ma.edu/

Please contact us if you have questions about any of these materials, or if you are interested in beta-testing portions of these learning guides in your classes. We have some money in our grant to support your beta-testing activities. In addition, early draft materials for learning guides on *Art and Sculpture, Patterns, Reasoning,* and *Calculus* on our web site, as well.

Thursday January 6, 2011, 2:15 p.m.-6:10 p.m.
MAA Session on The Mathematics of Games and Puzzles, I
Grand Chenier Room, 5th Floor, Sheraton
3:15 p.m. *Discovering the Art of Mathematics: Straight-Cut Origami.*
Christine von Renesse*, **Volker Ecke**, Westfield State College

Friday January 7, 2011, 2:00 p.m.-4:00 p.m.
MAA Poster Session on Projects Supported by the NSF Division of Undergraduate Education
Napoleon A1-A3, 3rd Floor, Sheraton
Discovering the Art of Mathematics. **Julian F. Fleron***, **Philip K. Hotchkiss**, **Volker Ecke**, **Christine von Renesse**, Westfield State College

Sunday January 9, 2011, 1:00 p.m.-6:00 p.m.
MAA Session on Humanistic Mathematics, II Mardi Gras GH, 3rd Floor, Marriott
2:00 p.m. *Student Inquiry into the Limits of Knowledge - Removing Barriers in Mathematics for Liberal Arts*.
Philip K Hotchkiss*, **Julian F Fleron**, **Volker Ecke**, **Christine von Renessee**, Westfield State College

Sign-up: If you would like to be kept up-to-date on the release of new materials, please add your name and email address to the sign-up sheet. We'll be happy to send occasional information about significant updates.

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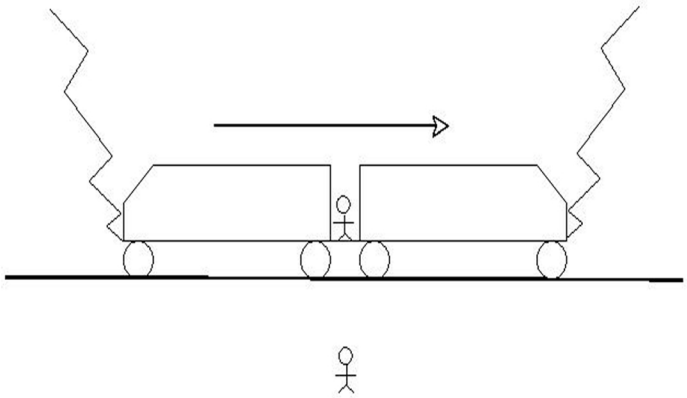
[1] These materials are also based on work supported by Project PRIME which was made possible by a generous gift from Mr. Harry Lucas.

Einsten's Theory of Special Relativity Example in an Inquiry Based setting

The next example is directly from Einstein's Relativity: The Special and General Theory.

Suppose a conductor is standing at the exact middle of a long, fast-moving train. A hiker is standing still, next to the tracks, watching the train speed by. Simultaneously flashes of light reach the hiker from bolts of lightning which have hit the very front and very back of the train. The bolts of lightning have left marks on the ground and it is later determined that the hiker is standing midway between the lightning strikes.



22. Explain why, from the perspective of the hiker, the bolts of lightning must have hit the train simultaneously.

23. As judged by the hiker, when the bolts of lightning hit the train, where must the conductor have been in relation to the hiker? Explain.

24. In the time it takes for the light from the lightning flashes to travel to the conductor, how has the location of the conductor changed?

25. Which flash of lightning will the conductor see first? Explain.

26. Immediately following this thought experiment, Einstein concludes:

> Every reference-body (co-ordinate system) has its own particular time ; unless we are told the reference-body to which the statement of time refers, there is no meaning in a statement of the time of an event.

Explain this in the context of our thought experiment.

27. What implications does this ***relativity of time*** have for our discussion of the limits of knowledge? Explain.

Discuss these questions with your neighbors!

11. What do we need to know about p and q in order for the conditional statement, "If p then q" to be false?

12. If we believe the conditional statement, "If p then q" to be true, and we believe p to be true, what can we conclude about q?

13. If we believe the conditional statement, "If p then q" to be true, and we believe q to be false, what can we conclude about p?

Now let us return to Smullyan Island and explore some of the subtleties of conditional statements on the Island.

14. Can a Knave say, "If I am a Knight, then there is gold on Smullyan Island?" Explain.

15. Can a Knave say, "If I am a Knight, then 1+1=3?" Explain.

16. Can a Knave make any statement of the form, "If I am a Knight, then q," for any statement q? Explain.

17. Can a Knight say, "If I am a Knight, then there is gold on Smullyan Island" if there is no gold on the island? Explain.

18. Can a Knight say, "If I am a Knight, then 1+1=3?" Explain.

19. Can a Knight say, "If I am a Knight, then q" for any false statement q? Explain.

20. Suppose a native of Smullyan Island makes a statement of the form "If I am a Knight, then q" for some statement q; based on your answers to Investigations **14-19**, what can you conclude about the native and the truth or falseness of q? Explain.

21. Suppose you meet a native who you believe is a Knight, and the native says, "There is gold on Smullyan Island." Do you go searching for gold? Explain.

22. Suppose you meet a native who you believe is a Knave, and the native says, "There is gold on Smullyan Island." Do you go searching for gold? Explain.

23. Can any inhabitant of Smullyan Island ever say "I am not a Knight."? Explain.

24. Suppose you meet a native who you believe is a Knight, and the native says, "You will believe that I have a pot of gold." What do you believe? Explain.

25. Suppose you meet a native who you believe is a Knight, and the native says, "You will never believe that I have a pot of gold." What do you believe? Explain.

26. Suppose you meet a native who you believe is a Knight, explain why you believe whatever the native says.

Our investigation of Gödel's Incompleteness Theorems will revolve around a native of Smullyan Island making the statement "You will never believe I am a knight." In order to completely examine this scenario, we need to examine how various reasoners will deal with this situation. Our first case will deal with the situation in which the reasoner, whom we will call Raymond.*, believes he is incapable of making a mistake. That is, if Raymond believes a particular statement, then, as far as he is concerned, the statement is really true. For example, if Raymond ever believes a native of Smullyan Island is a Knight, then because he thinks he is incapable of making mistakes, he concludes that the native really is a Knight and therefore everything that person says is true.

27. Suppose the reasoner, Raymond, believes that he is incapable of making a mistake. He meets a native who says, "You will never believe that I am a Knight." Suppose Raymond begins with the case that the native is a Knave. What can he conclude from the native's statement? Explain.

28. Based on your answer to Investigation **27** and the fact that Raymond thinks he is incapable of making mistakes, what does he conclude the native to be? Explain.

29. Explain why your answer to Investigation **28** gives rise to a contradiction.

*We need to use a third party as the reasoner because to reach the appropriate conclusions you, the student, need to be "outside" the system.

CHAPTER 1

A Sound of Thunder

Read the Ray Bradbury short story *A Sound of Thunder* which can be found at the URL http://www.scaryforkids.com/a-sound-of-thunder/ .

1. What did you think of the story?

2. Do you believe that going back in time to the Jurassic Period and killing a butterfly could really change the present in a significant way as proposed by Bradbury? Explain.

Bradbury's use of the butterfly as the catalyst for changing history was prescient. In 1972 Edward Lorenz used the term butterfly in the title of a talk on his seminal result about *sensitive dependence on initial conditions*, which is now commonly referred to as the ***butterfly effect***. In the next few questions we will explore this phenomenon.

3. Consider the rule $x \to \dfrac{2x^3}{3x^2-1}$. Plug in $x = 0.55$ into the rule, what is your answer?

4. Plug your answer to Investigation **3** back into the rule $x \to \dfrac{2x^3}{3x^2-1}$, what is your answer?

5. Plug your answer to Investigation **4** back into the rule $x \to \dfrac{2x^3}{3x^2-1}$, what is your answer?

6. Repeat the process from Investigation **5** several times until you see a pattern forming, describe the pattern.

7. Repeat Investigations **3-6** except now start with $x = 0.58$.

8. Compare the patterns from Investigations **6-7**, and describe how they changed as the value of x changes from 0.55 to 0.58.

9. Can small changes make big differences in long term behavior? Explain.

10. In light of your answer to Investigation **9**, could the death of a butterfly in the past make big changes to the future? Explain.

11. What implications do these Investigations have for our discussion on the limitations of knowledge? Explain.