Almost everyone knows that mathematics serves the very practical purpose of dictating engineering design. ... It is ... less widely known that mathematics has determined the direction and content of philosophical thought, has destroyed and rebuilt religious doctrine, has supplied substance to economics and political theories, has fashioned major painting, musical, architectural, and literary styles, has fathered our logic, and has furnished the best answers we have to fundamental questions about the nature of [humans] and [our] universe.
Morris Kline
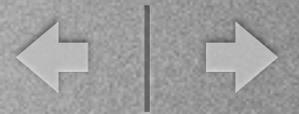
Student Inquiry into the Limits of Knowledge - Removing Barriers in Mathematics for Liberal Arts

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As the twentieth century draws to a close, it has become increasingly clear that Gödel's famous Incompleteness Theorem for mathematical logic stands alongside Heisenberg's Uncertainty Principle and Einstein's Theory of Relativity as one of the great mathematical achievements of this - or any other - century. Indeed, ... each demonstrates, in its own peculiar way, a kind of limitation in principle of the relevant formal science.

Palle Yourgrau- from <u>Gödel Meets Einstein: Time Travel in the Gödel Universe</u>

Einstein's Theory of Special Relativity

This example is directly from <u>Einstein's Relativity: The Special and General Theory</u>.

Suppose a conductor is standing at the exact middle of a long, fast-moving train. A hiker is standing still, next to the tracks, watching the train speed by. Simultaneously flashes of light reach the hiker from bolts of lightning which have hit the very front and very back of the train. The bolts of lightning have left marks on the ground and it is later determined that the hiker is standing midway between the lightning strikes.

Einstein's Thought Experiment on Relativity
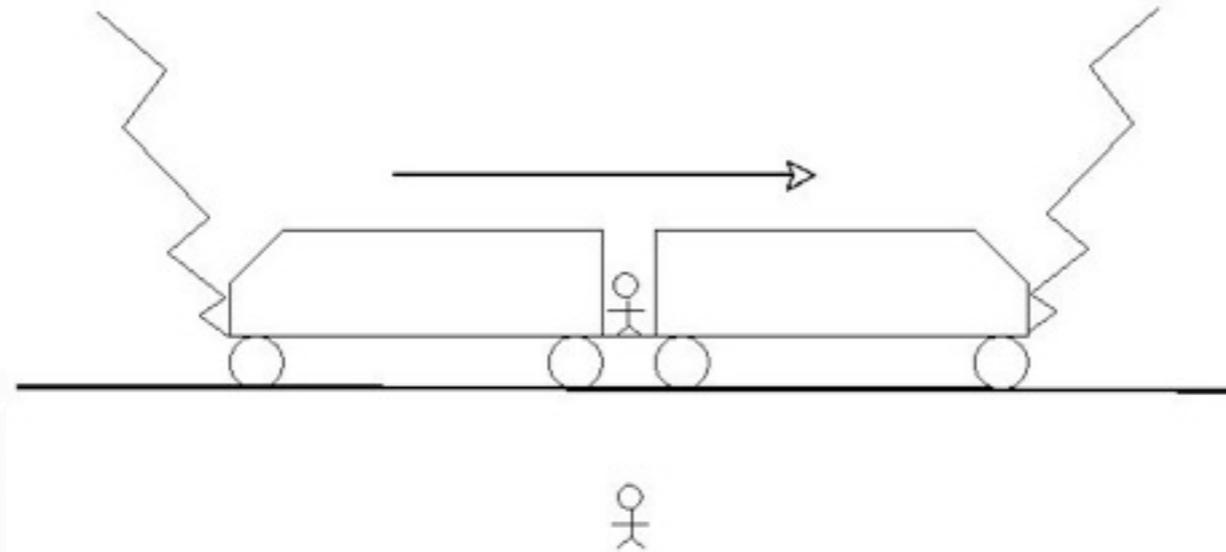
22. Explain why, from the perspective of the hiker, the bolts of lightning must have hit the train simultaneously.

24. In the time it takes for the light from the lightning flashes to travel to the conductor, how has the location of the conductor changed?

25. Which flash of lightning will the conductor see first? Explain.

26. Immediately following this thought experiment, Einstein concludes:

> Every reference-body (co-ordinate system) has its own particular time ; unless we are told the reference-body to which the statement of time refers, there is no meaning in a statement of the time of an event.

Explain this in the context of our thought experiment.

27. What implications does this ***relativity of time*** have for our discussion of the limits of knowledge? Explain.

Gödel's Incompleteness Theorems

- We use Raymond Smullyan's Knights and Knaves approach form his book <u>Forever Undecided</u>.

  - For those of you unfamiliar with this, Smullyan's Knights and Knaves are natives of an island where every person is either a Knight, who always tells the truth; or a Knave, who always lies.

14. Can a Knave say, "If I am a Knight, then there is gold on Smullyan Island?" Explain.

17. Can a Knight say, "If I am a Knight, then there is gold on Smullyan Island" if there is no gold on the island? Explain.

23. Can any inhabitant of Smullyan Island ever say "I am not a Knight."? Explain.

26. Suppose you meet a native who you believe is a Knight, explain why you believe whatever the native says.

Of Knights, Knaves and Gödel

- After running through some basic Knights and Knaves logic problems like those we just saw, we lead the students through an investigation of the statement, "You will never believe that I am a Knight" which is made by a native.

  - Though the work is difficult, the students eventually reach the conclusion that they will never believe the native is a Knight and that they will also never believe that the native is a Knave.

  - The native's statement models the unprovable statement in Gödel's First Incompleteness Theorem.

Sensitive dependence on initial conditions

- Another investigation of certainty uses the Ray Bradbury short story *A Sound of Thunder*, which can be found online at http://www.scaryforkids.com/a-sound-of-thunder/ . This story leads us into a discussion of sensitive dependence on initial conditions.
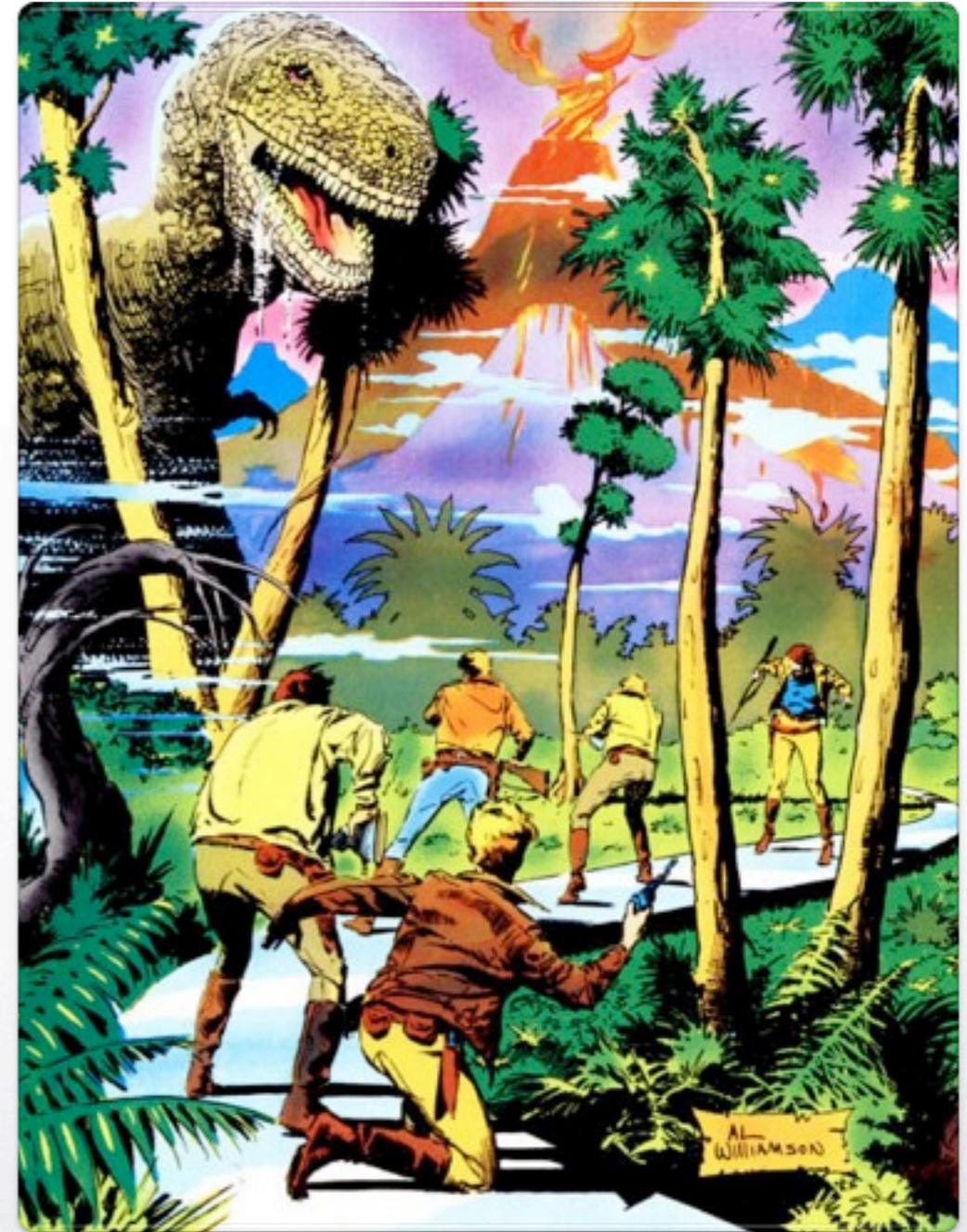
Read the Ray Bradbury short story *A Sound of Thunder* which can be found at the URL http://www.scaryforkids.com/a-sound-of-thunder/ .

1. What did you think of the story?
2. Do you believe that going back in time to the Jurassic Period and killing a butterfly could really change the present in a significant way as proposed by Bradbury? Explain.

Bradbury's use of the butterfly as the catalyst for changing history was prescient. In 1972 Edward Lorenz used the term butterfly in the title of a talk on his seminal result about *sensitive dependence on initial conditions*, which is now commonly referred to as the **butterfly effect**. In the next few questions we will explore this phenomenon.

3. Consider the rule $x \to \dfrac{2x^3}{3x^2 - 1}$. Plug in $x = 0.55$ into the rule, what is your answer?

4. Plug your answer to Investigation **3** back into the rule $x \to \dfrac{2x^3}{3x^2 - 1}$, what is your answer?

5. Plug your answer to Investigation **4** back into the rule $x \to \dfrac{2x^3}{3x^2 - 1}$, what is your answer?

6. Repeat the process from Investigation **5** several times until you see a pattern forming, describe the pattern.

Discovering the Art of Mathematics

http://artofmathematics.wsc.ma.edu

- This project is developing 10 Inquiry-Based Mathematics for Liberal Arts books

- Our books include
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  - Games and Puzzles
  - Art and Sculpture
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  - Calculus
  - Reasoning, Truth, Logic and Certainty
  - The Infinite

Beta-Testing and Reviewing

- We are currently looking for beta-testers and reviewers for some of our materials.

- If you are interested please contact me: photchkiss@wsc.ma.edu or Julian Fleron jfleron@wsc.ma.edu

References

1. Bradbury, Ray, <u>A Sound of Thunder</u>, 1952, Collier's Magazine. Available online at http://www.scaryforkids.com/a-sound-of-thunder/

2. Einstein, Albert, <u>Einstein's Relativity: The Special and General Theory</u>, Dover Publications, 2010

3. Naliboff, Jane and Hopkins, Jeff, <u>The Only One Club</u>, Flashlight Press, 2004

4. Smullyan, Raymond, <u>Forever Undecided: A Puzzle Guide to Gödel</u>, Oxford Paperbacks, 2000.