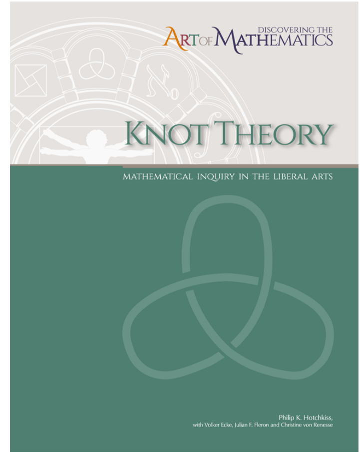
The following is excerpted from:

# Discovering the Art of Mathematics: Knot Theory

By Philip K. Hotchkiss, with Volker Ecke, Julian F. Fleron and Christine von Renesse




As with all of our learning guides, this book is freely available online at

`http://www.artofmathematics.org/books/`

*Discovering the Art of Mathematics* (DAoM) is an NSF supported project that supports inquiry-based learning (IBL) approaches for mathematics for liberal arts (MLA) courses.

The DAoM curriculum consists of a library of 11 inquiry-based learning guides. Each volume is built around deep mathematical topics and provides materials which can be used as content for a semester-long, themed course. These materials replace the typical lecture dynamic by being built on inquiry-based investigations, tasks, experiments, constructions, data collection and discussions.

DAoM also provides a wealth of resources for mathematics faculty to help transform their courses. Extensive online resources include volume specific teacher notes and sample solutions, classroom videos of IBL in action, sample student work, regular blogs about teaching using IBL and a regular newsletter. Opportunities for supported reviewing and beta testing are also available.

For departments interested in IBL, DAoM offers traveling professional development workshops.

Full information about the *Discovering the Art of Mathematics* project is available at

`http://www.artofmathematics.org`

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Use the excerpt below to explore for yourself how our materials can engage students in mathematical inquiry.

### Counting planar Tangles®

*The main tool for this course is a mathematical toy called a Tangle®. Tangles® are made from little plastic pieces that form a quarter circle and can be snapped together.*

*After exploring the different simple closed loops that can be made with Tangles®, the students observe that when the number of Tangle® pieces is a multiple of 4, i.e. 4, 8, 12, etc, they can be made to lie flat on the table.*

We call these ***planar Tangles***®, and we say two planar Tangles® are **geometrically distinct** if we can not change one planar Tangle® into the other by using *rigid motions*; that is by simply rotating or flipping the entire Tangle®.

**1.** For each of the first four number of pieces that make planar Tangles®, determine the number of geometrically distinct planar Tangles® shapes that can be made. Either sketch or use you cell phone to take a picture of each of these shapes to include in your notebook/write-up.

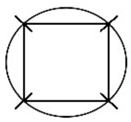
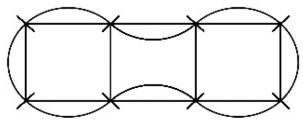
**2.** Do you think there is a recognizable pattern in the number of geometrically distinct planar Tangles® shapes? Explain.

**3.** At this point things get more complicated. After the planar Tangles® you considered in Investigation **1**, the next smallest planar Tangle® has 31 geometrically distinct shapes. Find as many of them as you can and include a picture of each shape.

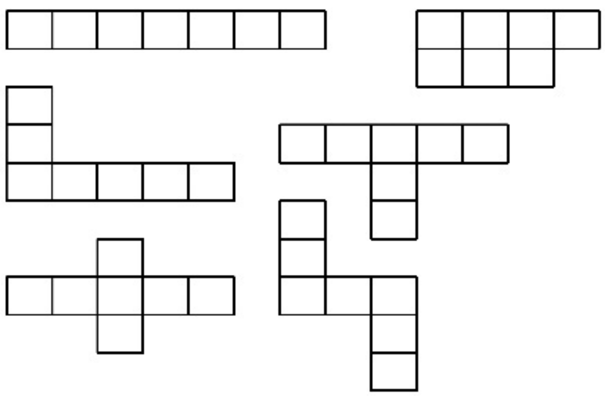
To simplify our writing in what follows, we will call any Tangle® made from $n$ pieces an **n-Tangle®**.

We can use graph paper to draw each planar Tangle® by using the outside edges of the squares to guide our drawing of each Tangle® piece. The main requirement is that the quarter circles match up correctly as they would in a planar Tangle®.

Here are two examples. We can match the planar 4-Tangle® to a single square and the 8-Tangle® to a three squares as follows.


**4.** In the figure below there are six shapes made up from seven squares which we call ***7-polyominos***. Some match up to planar $n$-Tangles® you made for a specific value of $n$. Draw in the Tangles® to determine which shapes match up to $n$-Tangles®. What is this value of $n$? Why do some of the shapes match up and not others?



**5.** Do all 16-Tangles®correspond to 7-polyominoes? Explain.

It turns out that finding a formula that gives the number of planar $n$-Tangles® for each value of n is difficult. In fact, it is unclear whether this mathematical problem has been solved, and **Julian Fleron** (American Mathematician; 1966 - ), has conjectured that this is related to a very important and unsolved problem in another area of mathematics, the ***Polyomino Enumeration Problem***. This is a very old problem that essentially tries to count the number of distinct **polyominos**, connected figures one can make with $n$ squares with the requirement that each square share at least one side with another square. For example, the figures above are polyominos made from seven squares. Since these are made with seven squares, we call these **seven-polyominos**.

Some of the above mentioned polyominos may look familiar to you if you have ever played the computer game Tetris. This (seemingly) simple game has a connection to another very important unsolved problem, the *P vs. NP* problem, which has a $1,000,000 prize for its solution offered by the Clay Mathematics Institute. (To find out more about this connection, visit our website and download Discovering the Art of Mathematics: Knot Theory.)