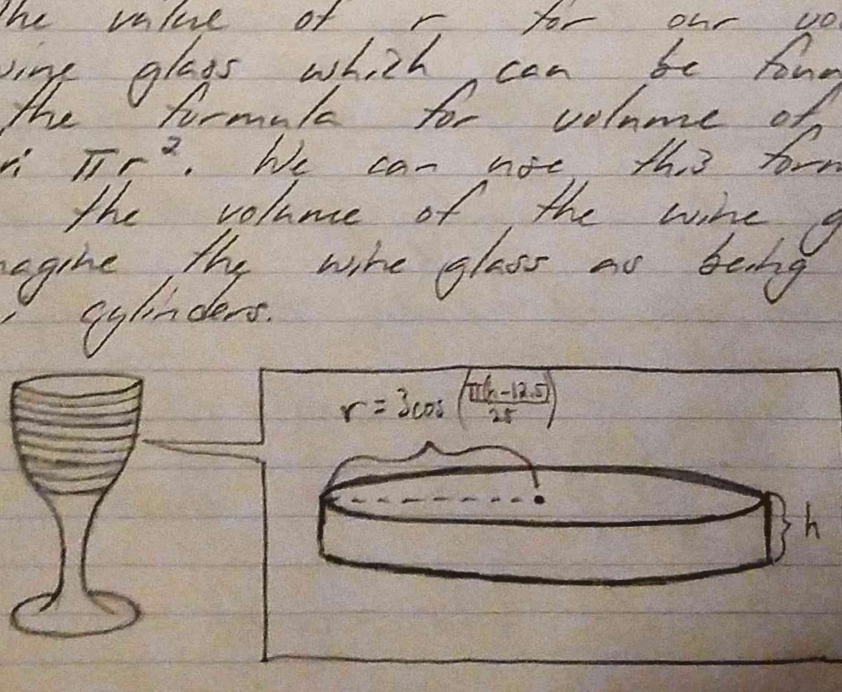
Morgan Shia 3/27/18 Story: Wineglass Integral

For this assignment we were asked to calculate the volume (both approximately in the form of a Riemann sum and exactly in the form of an Integral) of a wine glass given the following information:

"The radius, r, is a function of the height, h, and given by $r = 3\cos\left(\frac{\pi(h-12.5)}{25}\right)$ where $0 \leq h \leq 14$."

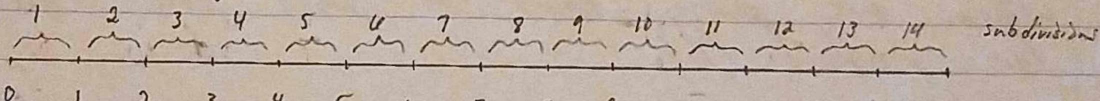
In other words, our radius at a specific height (h) equals whatever number we calculate when we plug that "h" into the equation for radius (r).

This is important to realize because we'll need the value of r for our volume of the wine glass which can be found by using the formula for volume of a cylinder: πr^2. We can use this formula to compute the volume of the wine glass if we imagine the wine glass as being made up of mini cylinders.



So now that we know how to find the volume of a cylinder, and we know the TOTAL volume is the sum of all of those cylinders, we can try and come up with a Riemann sum, which would allow us to approximate the total volume by adding up every individual cylinder at one time instead of having to compute each on their own.

Given that the wine glass' heights are between 0 and 14, we can conclude that if each cylinder has a height of 1 cm (I forgot to mention earlier our units are cm), then there must be 14 subdivisions/cylinders.



(Wow... that's a huge wine glass! Hahaha)

Since there are 14 subdivisions, our starting point can either be 0 or 1, but if we start at 0 our ending point should be term 13 and if we start at 1, our ending term should be 14 to ensure we're adding up 14 terms.

$$V = \sum_{h=1}^{14} \pi \left(r(h)\right)^2 \cdot 1$$

$r(h)$ = radius as it changes with corresponding heights

$\pi \cdot r^2 \cdot h$ (of each cylinder)

Using that Riemann sum, and plugging it into Desmos I found that $V \approx 232.14 \, cm^3/mL$.

Keeping in mind that is just an approximation, we were asked to create another Riemann sum that is more accurate. To do that we need more subdivisions, so if we take the height of 1 cm and divide it into 2 smaller sections each .5 cm high. Since we created 2 sections from what used to be 1 section, we are essentially doubling the total number of subdivisions. But because we are changing the height (by dividing it by 2) we also need to change what follows the Riemann sum.

$$V = \sum_{h=1}^{28} \pi \left(r\left(\tfrac{h}{2}\right)\right)^2 \cdot \frac{1}{2}$$

← Multiplied by $\frac{1}{2}$ because $\frac{1}{2}$ is our cylinders' new height.

Our height here is divided by 2 since we are inputting "half-heights" to our radius equation.

$$\underline{\underline{V \approx 225.42 \, cm^3/mL}}$$

Since our ultimate goal is to find an integral which will give us the exact volume of the wine glass, it is important to make connections between our Riemann sums and the general forms of both the Riemann sum and an Integral to see how they're related.

The following are the general forms of the Riemann sum and Integral as they relate to each other.

$$\lim_{n\to\infty} \sum_{k=1}^{n} \overbrace{f\left(a+k\left(\frac{b-a}{n}\right)\right)}^{f(x)} \cdot \overbrace{\frac{b-a}{n}}^{\text{this is separate!}} \qquad \int_a^b \overbrace{f(x)}^{f(x)}\,dx$$

Now if we compare that to our Riemann sums we can match up the pieces.

$$V = \sum_{h=1}^{14} \overbrace{\pi(r(h))^2}^{f(x)} \cdot 1$$

$$V = \sum_{h=1}^{28} \overbrace{\pi\left(r\left(\frac{h}{2}\right)\right)^2}^{f(x)} \cdot \frac{1}{2}$$

$$\frac{b-a}{n}$$

$$\boxed{\begin{array}{c} \frac{14-0}{14} = \frac{14}{14} = 1 \\ \frac{14-0}{28} = \frac{14}{28} = \frac{1}{2} \end{array}}$$

b is our ending height,
a is our starting height,
and n is the number of subdivisions/cylinders.

Since we've found our f(x) in the Riemann sum we can now write our integral using 14 as our b, 0 as our a, and $\pi(r(h))^2$ as our f(x).

$$\int_0^{14} \pi(r(h))^2\,dh = 218.48\,cm^3/mL$$

It's getting hard to find decent math jokes so here are some riddles!

① A cowboy rides in on Monday, stays for two days, and then leaves on Friday... how is this possible?

② What parking spot is the car in?