

**Final Exam  
Calculus 2  
Spring 2018**

**Name:** \_\_\_\_\_

Please explain all your thinking in detail. You are not allowed to use a graphing calculator or any notes for the exam.

**Skills/Facts: (20 points)**

1. Write down the equation of the Taylor series of a function  $f(x)$  at  $x = a$ .
2. Compute the integral  $\int x e^x dx$ .

**Methods: (40 points)**

3. For which  $x$  does the following power series converge? Explain your thinking.

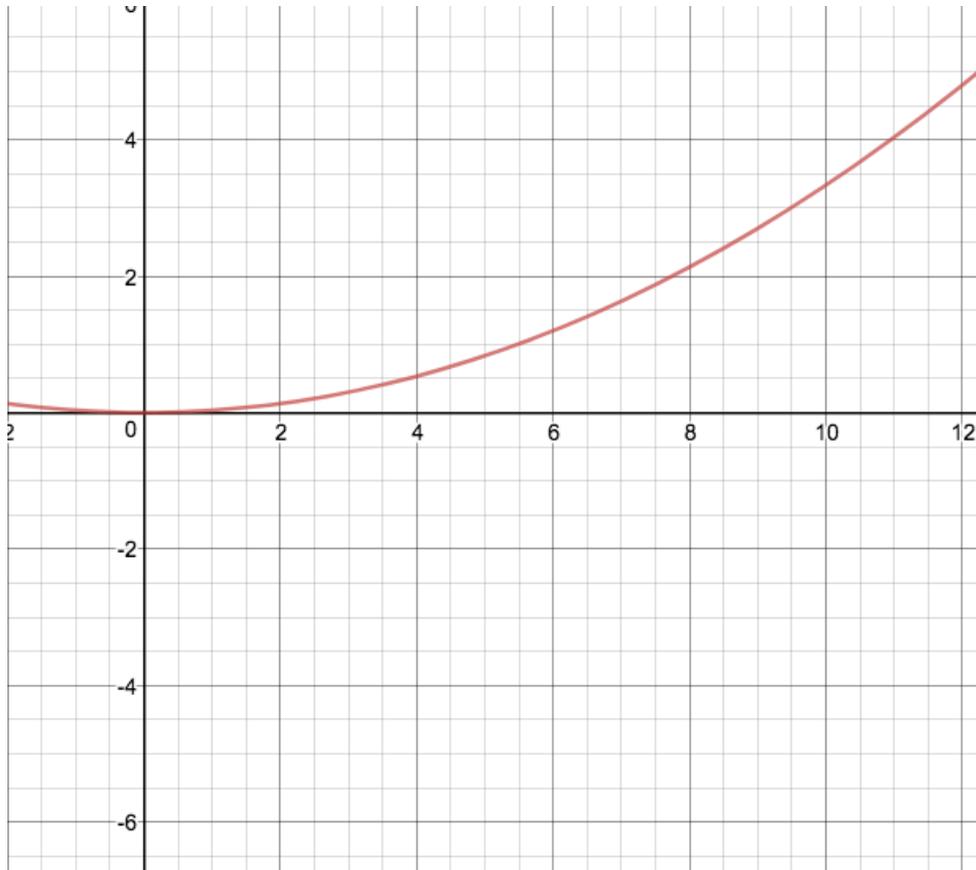
$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k 4^k} (x - 4)^k$$

4. Decide if the following integral is convergent or divergent. Explain your thinking.

$$\int_0^5 \frac{2}{(x-3)} dx$$

**Conceptual Understanding: (40 points)**

5. Shade the region between the red graph of the function  $g(x)$  and the x-axis and between  $x = 0$  and  $x = 10$ , in the following picture. Now rotate this region around the x-axis.



- What does the resulting 3-dimensional shape  $S$  look like? Draw a picture.
  - Explain how you would use 5 subdivisions to estimate the volume of  $S$ . (You don't have to actually estimate it)
  - Which integral would compute the volume of  $S$  precisely? (You don't have to actually compute it)
6. Use the example of the p-series  $\sum_{k=1}^{\infty} \frac{1}{k^2}$  to explain **why** the integral test works. Draw pictures to explain your thinking.

**Extra Credit:**

7. The following picture shows the convergence of a specific series.
- Which series is it?
  - Which value does the series have according to the picture?
  - Find a (different) computational way to compute the (same) value of the series.

