Day by day summary of Math 1300, Mathematical Explorations, Fall 2014,
Prof = Steven Strogatz, TA = Yao Liu

Day 1 – Aug 26

Welcome. Greet them at the door. Tell students: Write your name (first name big) on a folded big index card. That will be your nameplate throughout the class. Nameplates will be moved around randomly from one day to the next, from table to table, so that students meet each other and work together.

Introduce Professor and TA, say fine to call us by first names unless you prefer Professor.

Go through page 1 of the handout on "our IBL classroom" to explain what inquiry-based learning is, and how our class will be different from what they used to and why we are doing it.

Gallery walk: what are some words that describe math, what is the first feeling that comes to mind when they think of math, who are some famous mathematicians, why would someone want to study math, what other subjects are they interested in, etc. Followed by whole class discussion. Intended to let us get to know them, and for them to get to know each other. Also, to show that their attitudes are shared by many, and to acknowledge them.

Activity: Straight-cut origami. Examples from http://www.artofmathematics.org/books/art-and-sculpture. Folding and straight-cutting an equilateral triangle, square, 2 by 1 rectangle, 2 line segments that make equal angles with a third segment (like a V with a flat bottom instead of a corner). No discussion of the math yet, or what their strategies are. That will come in the next class.

Day 2 – Aug 28

Reviewed folding equilateral triangle and rectangle. Asked them to think about the lines in the successful fold patterns. Asked them to say what they "notice" and what they "wonder" about their results so far. They spoke of lines of symmetry, but still have not yet said "angle bisector."

Then gave them the scalene (irregular) triangle. Class spent at least 30 minutes on it. Only one student managed to solve it. But rest of the class was happily engaged. When I asked them if they wanted a hint, a few said yes, but were quickly drowned out by others who said no. That was a huge high point for me and for them. Remarkable!

Need to praise them for their stick-to-it-iveness next time.

Also, need to guide them toward the math behind the scenes. I have not passed out the worksheets yet. Maybe time to get them doing that and writing up their thoughts so far.

Gave them an assignment: write up answers to questions 19-24 in the book, submit them as journals in Blackboard, showing me their thought process, conclusions, arguments. I haven't figured out yet how to grade those journals. Need to learn how the Westfield St people do the grading.

Day 3 – Sep 2

Final session on folding and cutting. Began with double angle, had everyone do it again. Discussed the role of angle bisectors. Got them to notice two things about the other fold line: It's both perpendicular to the short side, and it bisects that side. (The bisector property turns out to be a red herring for later questions, but they don't know that yet.)

Then had students sit at tables, tracked by what they figured out over the weekend. Those who had not solved the scalene triangle sat at one set of tables. Those who had solved the triangle but not the quadrilateral sat at another. And those who'd done both got to try the non-convex quadrilateral.

Lots of squeals of delight at various times as people solved triangles for the first time. I let them spend a long time trying things, teaching each other.

Ended class by having them summarize what they learned about pinch folds, "mountain" and "valley" folds (as one Korean student who knew some origami called them), and the role of angle bisectors and perpendiculars.

Did not have time to state or discuss Erik Demaine's work and his big theorem (can do straight cut origami on any finite union of polygons).

No one solved the non-convex quadrilateral.

A few students sent me e-mail or Blackboard comments about what a transformative experience this activity was. E.g., From a student, on solving straight-cut origami task with scalene triangle: "I am feeling exceptionally accomplished. I have to admit: this math assignment has made my day. I never thought I would ever be saying this." From another: "I feel obligated to add a comment on my in-class progress. My original technique for solving the scalene triangle, despite my initial doubts, was in fact accurate. Moreover, I was successfully able to apply that to the irregular quadrilateral. This process of experimentation and persistence has real world application as it demonstrates the importance of continuing a task while simultaneously approaching it in new and unusual ways. I have already learned a great deal from this class while still enjoying math, a feat I thought impossible only two weeks earlier."

Day 4 – Sep 4

Started Chapter 1 of Dance. http://www.artofmathematics.org/books/dance
Began by having them list examples of symmetry – each table listed many examples at their board and then we shared them -- architecture, faces (Denzel Washington got a laugh; supposedly he has the most symmetrical face and that's why he's so handsome), music, art, animals, synchronized swimming, etc.

Did mirror game with body positions, dance moves, had each table work on answers to questions 1-11. Then whole class discussion.

Next, rotational symmetry. Same format as for mirror symmetry.

Played the switching symmetries game. They came up with the conjecture about when it's possible to switch smoothly between reflectional and rotational symmetry (it's possible when the pose is bilaterally symmetric). Next time, we'll try to prove it.

Day 5 – Sep 9

Spent the whole class on clarifying what the question is. Discussed what's needed to switch smoothly between reflectional and rotational symmetry. Pose must be bilaterally symmetric, with plane of reflectional symmetry through the body. Then simplified the problem to 2-D. Tried to get them to prove this: (no movement needed to switch) implies (bilaterally symmetric). Used a schematic with letters FJ (representing two halves of the body) reflected, and rotated, and showed J must equal backwards F. Some students got frustrated, bored. Seemed we were going too slowly, yet many were not understanding it. Some students later said they felt like we were going in circles.

Also, too much talk and abstract group work at their tables, not leavened by activity, standing, dancing, etc.

Day 6 – Sep 11

Great class today. Showed them what translational symmetry means (section 2.4 of chapter 1, Dance). They acted it out quickly, solved questions 32-35 in chapter 1. Showed them glide reflectional symmetry in the plane, and they acted it out. Then did the four symmetries on the line. Confusing point: mirror reflection M uses a line *perpendicular* to the given line (the line used for translation), whereas glide reflection uses the given line as the mirror. Spent a few minutes clarifying this and having them act out several examples and checking them. Next (and this was fun) we used groups of three students to act out what happens when one symmetry is followed by another. First student (the leader) strikes a pose; second student applies a symmetry; third student applies another symmetry to the pose of the second, then compares his or resulting pose to that of first student. Get results like MR = G, where M means mirror reflection perpendicular to the line, R = 180 degree rotation, G = glide reflection, i.e., reflection across the line and glide along it). Each table of students filled in the T,M,G,R multiplication table and found the Klein 4-group. With little prompting they noticed many cool patterns in the multiplication table. Finished by giving them a mini-lecture, mentioning larger significance of group theory in physics, etc. Next time: Frieze patterns.

Day 7 – Sep 16

Frieze patterns. Began with tracing paper activity. I had them make a frieze pattern by drawing a shape on white paper, then repeating it translationally along a line. Then trace that array and that line onto tracing paper. Flip, rotate, or translate that tracing onto the original so that the lines match up. Copy the result. This process generate friezes having various symmetry types: TV, TH, TG (where V and H mean reflections through vertical or horizontal lines). This activity was confusing to the students. I didn't give specific enough directions. Several of them made all the friezes on the tracing paper, which was not my intent. Need to improve this activity next time I try it.

Next, had them act out Conway's footstep patterns. Hopping, sidling, etc. Fun activity.

Then classify the various footstep patterns according to which symmetries they have: TVGHR. Had each table list all the symmetries for each pattern, and then compared them. Common confusion: they did not realize the line goes THROUGH the feet in spinning sidle. So they did not see G as a symmetry. Also, discussed the fact that we did not have H in the last class (and V was called M). Our classification is

Ended by having them figure out why there are only seven types. For example, why does TVG not appear in the list? Did not tell them how to approach it. Several tables realized R is implied by TVG. Good discussion of this, with various tables presenting their reasoning.

Day 8 – Sep 18
Pennies and paperclips: http://www.artofmathematics.org/blogs/jfleron/pennies-paperclip-proofs
Everything went as Julian said it would in his blog post. They quickly realized that Penny wins when pennies are on same color. All tables found proof pretty easily. Thought the same counting argument also worked for proving Clip wins with pennies on opposite color, but came to realize that was insufficient (since it ignores board geometry – some confusion about what the counterexample board really shows). Class ended with my mini-lecture and class discussion of what a Hamiltonian circuit is, and they tried to use it to give a proof for when Clip wins, but they did not manage it. I assigned it – and the write up for the easier case of when Penny wins – for homework.

Day 9 – Sep 23
Discussed Hamiltonian cycle. First on a loop checkerboard, then on 4 x 4 square checkerboard, to prove that paperclips win when the pennies are on opposite colors.

Explained that next several weeks will be about proof, reasoning, truth, etc. Start with "Doubt", chapter 1 of http://www.artofmathematics.org/books/truth-reasoning-certainty-and-proof

Spent rest of class slowly working through questions in sections 1.1 and 1.2. What are they certain of? Why? Had them discuss at each table, write answers on nearby blackboard, whole class discussion. Then did puzzles and discussed them: vanishing leprechaun, vanishing red and blue pencils, missing square in the triangle. Students loved the leprechauns especially, did not want to stop working on it.

Day 10 – Sep 25
Began by revisiting leprechauns. I asked the class to number the middle of the faces. Then, after rearrangement, saw one leprechaun got assigned two numbers; that's how a leprechaun disappeared. Some students were not convinced, or were more confused than before. Others felt they already understood it. And a few had Aha moments.

Continued our discussion of "Doubt", chapter 1 of http://www.artofmathematics.org/books/truth-reasoning-certainty-and-proof
Each table discussed three questions, then wrote responses on board near them, then had whole class discussions. Questions were:

48 (p.15) -- myths that have been debunked that you find interesting. Lots of fun responses to this one (5 second rule for food dropped on floor; cures for hangovers, etc.)

49 (p. 17) – beliefs that were widely accepted within a community and had outcomes that were damaging, dangerous or deplorable. Their responses were serious: Holocaust, drinking the Kool-Aid (Jim Jones), having sex with virgin as a cure for AIDS (a belief in some parts of Africa). Discussion got politically contentious when one table wrote "American Dream" and "capitalism." Another student objected, said those weren't comparable to Holocaust.

53 (p. 18) – things in math that are generally accepted as true but you don't necessarily believe them. Answers included disbelief in 0; doubts about pi; negative numbers; negative times negative equals positive; why isn't 1 a prime number? Class

begged me to answer questions about 1 as prime, and about negative times negative – I felt uncomfortable since it was not IBL style instruction, but they loved it and wanted more.

At the end of class, students were happy that discussion had gotten so heated about politics, thought this was great to discuss in math class (said Rebecca Allen). Also urged me to do more clarifying of basic math misconceptions or confusions. Another (Emma Woroch) said she has been telling her mother all the cool stuff she's learning in math class; mom says, "but you never liked math before!"

Day 11 – Sep 30

I started with a mini-lecture about Euclid's contribution to intellectual history, via the notions of an axiomatic system and deductive proof. Then launched into Newton, his influence on the Enlightenment, his use of Euclidean style in Principia, followed by a look at the Declaration of Independence as a Netwonian/Euclidean document.

Then, they wrote on boards: what do they know about pi? What do they wonder about pi?

Rest of class was about why pi, defined as C/(2r), is same for all circles. To get at that, we calculated pi-ish numbers for squares and hexagons of different sizes. (got 4 and 3, respectively). Class felt OK, but somewhat blah. Maybe too mathy for them? Or maybe just my own hangup, feeling the need to keep entertaining them with stuff that is more active and fun?

Day 12 – Oct 2

Had them cut out 8 and then 16 colored sectors of circle, reassemble them to an approximate rectangle, use that to explain why A = pi r^2. Class liked this. Got it. Briefly discussed that shape becomes a perfect rectangle in the limit of infinitely many infinitesimally thin sectors.

Then mentioned Euclid does not show this, but Archimedes did, and mentioned that he was less afraid of infinity. Briefly mentioned Zeno, and going halfway to the wall. That got us derailed for 10 minutes, discussing .9 repeating = 1, but class enjoyed it. Many doubters or disbelievers. One student proved it via 1/9 = .1 repeating. Another saw where he was going and said, "I see it now, but that's wrong!"

Then did activities about taxicab geometry, both discrete and continuous. (Next time, need to be clearer about what the book calls "continuous taxicab geometry". Just say that the distance to a point (x,y) is defined as abs(x) + abs(y).) Had them define circles, then draw them on graph paper. Asked them to figure out circumference of circles of radius 1,2,3, then of radius r. Area of circle of radius r (where 1 square block = 1 unit of area). Find C = 8r, and A = 2r^2. So area pi and perimeter pi are different in this geometry.

Felt like a good class. A few may not have gotten everything about taxicab geometry. (Some had sqrt(2) in circumference formula at first.) But most got it right.

Day 13 – Oct 7

Inductive reasoning, chapter 4 in http://www.artofmathematics.org/books/truth-reasoning-certainty-and-proof. Went straight through examples in the book. Played "What's My World?" Discussed its relation to the scientific method, reasoning from examples to rules.

Next, the list of presidents. Several students thought the pattern was that they all died in office. But one student, John Hall, pointed out that Zachary Taylor was missing from the list. The right rule was that these presidents were all elected in a year divisible by 20, *and* they died in office. "Tecumseh's Curse" (if you're elected president in a year ending in 0, you'll die in office) held until Reagan in 1980.

Next discussed list of arithmetic equations, tried to guess the next ones, briefly discussed Goldbach's conjecture. Also, why isn't 2+2=4 on the list?

Ended with a logic puzzle brought up by Sasha Kawasaki, about four prisoner's buried in sand up to their necks, each wearing a hat. Given: Two white, two black hats. Prisoners not allowed to speak, except for one to announce color of his own hat and to explain his reasoning.

Day 14 – Oct 9

Deductive reasoning, sec. 4.2 in http://www.artofmathematics.org/books/truth-reasoning-certainty-and-proof.
Knights and knaves logic puzzle. Sudoku as an example of logical deduction.
Finite geometries (three point and four point). Students were confused by some of the phrasing of the axioms in the book. Next time, say "For any two (distinct) points, there is exactly one line connecting them" and also, "Any two (distinct) lines have at least one point in common." Saying "in common" makes it clear for them.

Day 15 – Oct 16

Cheez-it math. Brought in paper towels (to avoid greasing up the tabletops, and also for hand cleaning) and 4 boxes of cheez-it crackers. Really fun class about proofs without words. Discussed odd and even, how to represent numbers as patterns of cheez-it crackers, then moved on to sums like 1+2+3+4+3+2+1, and 1+3+5+7, and 1+2+…+n. Asked them to make conjectures, then use the crackers to prove them. Several students spoke up who rarely do (Rush, Yujin, Rebecca), and gave good proofs at the board. Very exciting!

Ended with first magic trick in sec 5.3, asked them to figure it out over the weekend.

Day 16 – Oct 21

Visit from Linda Glaser (writer, College of Arts and Sci) and Jason (photographer). Had a great class about five ways to prove the sum of the interior angles of a triangle is a straight angle. Led the class through proofs by tearing, folding, walking, squinting, and tiling. We did not get to the tiling proof. Yujin showed the class the standard proof in Euclid.

Day 17 – Oct 23

Fibonacci numbers – first chapter in Number Theory book.
http://www.artofmathematics.org/books/number-theory

Counted the number of spirals (in both clockwise and counterclockwise senses) on sunflower heads and pinecones. Then worked through Fibonacci's rabbit problem, and tried to see why Fibonacci numbers arise there.

Class felt flat today. They were very quiet while constructing the "rabbit pair" family tree. But maybe they were busy calculating? That's what they said when I asked them. And when I pushed them to write down recursion formulas for the number of adult and baby rabbits in each month, they acted like it was unnecessary. They already were convinced that the numbers would be Fibonacci numbers, based on what they saw in the first few months. Out of frustration, I called them "gullible," and then immediately regretted it.

Day 18 – Oct 28

Golden ratio. Passed out first 8 pages from chap 2 of number theory book.
http://www.artofmathematics.org/books/number-theory
Started with asking them what they know or have heard about golden ratio (they wrote answers on board near them). Discussed interesting replies (e.g. golden ratio used in Catholic devotionals, Italian renaissance art, etc.) Had them measure their belly button heights and compute golden ratio for their bodies. To do this, brought a tape measure, had my TA mark off 40-50 inch markings on various whiteboards around the room, student walked up to local board and measured themselves. Looked at avg of all, was close to 1.63, range 1.55-1.72. Did nested radical and infinite continued fraction activities. They were rusty on quadratic formula, but were fast at seeing x^2 = 1+x. Next time I promised to bring in celebrity photos to see if golden ratio is really in pretty faces. Also, we can discuss claims in DaVinci Code.

Day 19 – Oct 30

Golden ratio in faces. Before class, assigned the students to watch this video
https://www.youtube.com/watch?v=kKWV-uU_SoI
and read this website:
http://www.goldennumber.net/golden-ratio-myth/
Asked them to read Devlin's skeptical MAA article:
http://www.maa.org/external_archive/devlin/devlin_05_07.html

In class, we read aloud together from DaVinci Code, by going around the room like in church, reading all the claims about PHI in art, nature, our bodies, etc. Then passed out images of celebrities: Denzel Washington, Jennifer Lawrence, Zachary Quinto, Angela Bassett, Kristen Wiig, Kim Jong Un. Measured the faces for alleged golden ratio. Surprising amount of measurement variability on the same image of a given face – ambiguity about where to draw the lines? Imprecise use of ruler in doing the measurements? Ended with discussion of honeybee math. Dan Brown says female/male ratio is golden. But does he mean ratio of female/male ancestors of a given male of female? And aren't we counting the queen again and again, so we really mean ancestors with multiplicity? Fun class.

Day 20 – Nov 2

String art. Chap 4 of Calculus book. http://www.artofmathematics.org/books/calculus

Brought in two string art models from library. From top view, one shows an ellipse, the other shows a cardioid. Brought in a circle with tick marks around it. Had them draw lines between marks at 10 and 20 deg, 20 and 40, 30 and 60, etc. all around circle. Envelope is a cardioid. Where is the cusp?

Measure it. Any explanation? (Think about line from 181 degrees. Where does it end up? At 362 deg = 2 deg. So 1 degree angle change produces 2 deg change on other side. Hence line must go through a point 1/3 of a diameter across, i.e., lever arms of see saw through the cusp must be in a ratio of 1 to 2. One student showed this with similar triangles.) Showed cardioid formed by light reflecting in a coffee cup. Can you explain that? Light bounces around rim – hits at angle x, bounces off to mark at angle 2x (this is equivalent to usual law of reflection. Here, think of light as billiard ball, always moves through same arc along rim of circle. Where many of the rays intersect you get a caustic, hence bright spot. That happens on envelope of the lines, which is the cardioid.) One of the students mentioned cardioid microphone – we discussed what that is, and how the cardioid quantifies its directional pickup. Another student mentioned cardioid in the Mandelbrot set. Fun class today!

Day 21 – Nov 6
Area and Koch curve. Looked at chapter 1 in Calculus book.
http://www.artofmathematics.org/books/calculus
Tables wrote on boards where area calculations are needed in real life. Discussed why *xy* make sense as the right formula for area of rectangle. Considered three properties that area should have (additivity, invariance under rigid motion, area of unit square =1) and showed that (x^2)y would not work. Then found areas for various simple shapes on geoboards. Ended by trying to find area inside Koch snowflake. Class did not finish. We'll continue next time, using same teams at tables.

Day 22 – Nov 11
After quickly finishing the area of the Koch curve, we discussed Lockhart's *A Mathematician's Lament*. Gave a four-question quiz on it (I announced it the day before; gave it mainly to make sure everyone read the piece and was ready to discuss it.) Really fun discussion. Class didn't want to stop – lots of hands up, offering to speak, at all times. Article about our class in the Cornell Chronicle today:
http://www.news.cornell.edu/stories/2014/11/strogatz-helps-students-find-magic-math

Day 23 – Nov 13
We tried the 3a+5b activity. From "student toolbox" book at
http://www.artofmathematics.org Also lots of great examples of Westfield State student work at: http://artofmathematics.org/blogs/jfleron/3a5b-proofs

Day 24 – Nov 18
Sec 1.2 in "The Infinite" book. http://www.artofmathematics.org/books/the-infinite Big numbers in everyday life – did lots of the questions in sec 1.2. On Internet, they looked up what you can buy for a thousand, million, billion, or trillion dollars. Calculated how

long 1 million vs 1 billion seconds is (12 days vs 32 years). Discussed Terry Tao's post on rescaling the federal budget.

Day 25 – Nov 20
Chapter 5 from "Games and Puzzles" – http://www.artofmathematics.org/books/games-and-puzzles One pile, two piles, three piles. (aka Nim). Brought in M&Ms, Skittles, Cheez-Its, and Smarties (plus paper towels). Class loved this activity. They figured out the one pile and two pile games. We did not have time to get to three piles.

Day 26 – Dec 2
The Human Knot game. Sec 6.2 of Knot Theory Book.
http://www.artofmathematics.org/books/knot-theory
Brought string, scotch tape, scissors to class. Yao (the TA) and I demonstrated that with two people, you can't make a knot. Had them try it with three people. They untangled every time. Then many of them started to do four people, also untangled most of those. Some tables spontaneously joined and did much bigger knots. They were having a blast doing these exercises.

Then I told them a bit about knot theory. Asked them who uses knots in real life. They shouted out: sailors, weavers, mountain climbers. I told them about knots in science. Vortex atom theory of chemistry. Knots in DNA. Knots on string theory. (Olivia asked, "What are quarks?" which brought me back to earth and reminded me who I'm talking to).

I told them about knot tables, passed outs table of knots up to 7 crossings. Had them make a human trefoil knot. That challenged them, they liked it. Many of them next tried to make figure 8 knot, but it's a bit intimate, some students decided they did not want to be part of it. At which point I had them make string models of figure 8 knot, but that did not work well. Apparently too hard to visualize. (?) Anyway, I asked them to see if they could deform figure 8 knot to its mirror image, which turned out to be way too hard for them. Most could not even make the knot let alone its mirror image. Next time, get Tangles® and make knots with them.

Overall, a fun class, but need to rethink the end.

Day 27 – Dec 4
Gallery walk about what they now think of math (compare to responses on day 1), and what activities they liked and did not like. End with my mini-lectures on anything they wonder about it math.