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Math Explorations
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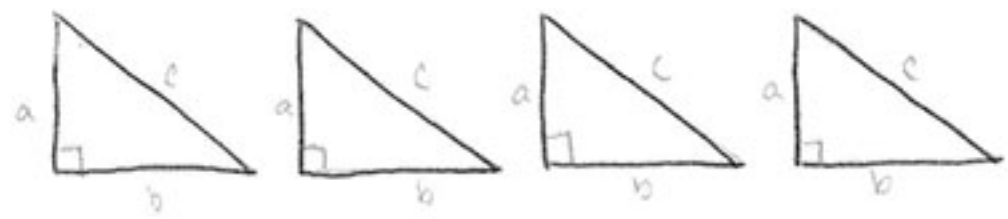
The Pythagorean theorem

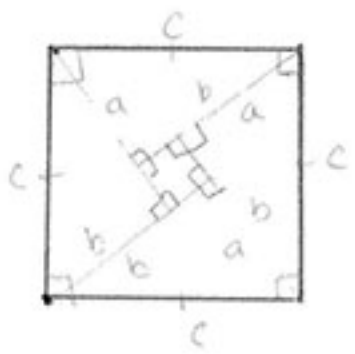
Conjecture: In a right triangle with sides a, b, and hypotenuse c:

$$A^2 + B^2 = C^2$$

Proof: If you have four identical right triangles:

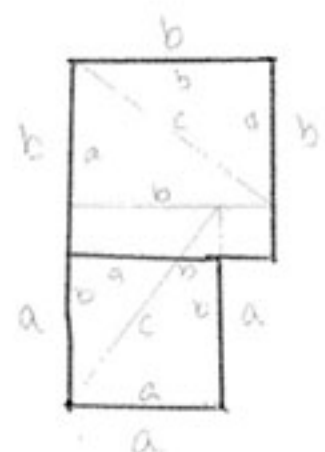


Arrange them so the hypotenuses, or side's C, of the four triangles makes four sides of a square:



You know the shape above is a square because all of the sides are equal and the angles are all 90°. We know this because every triangle is 180° and since they are right triangles one of the angles is 90° and the other two add up to 90° and the other two angles happen to make one angle of the square. Also, we know the sides are equal because they are all hypotenuse C.

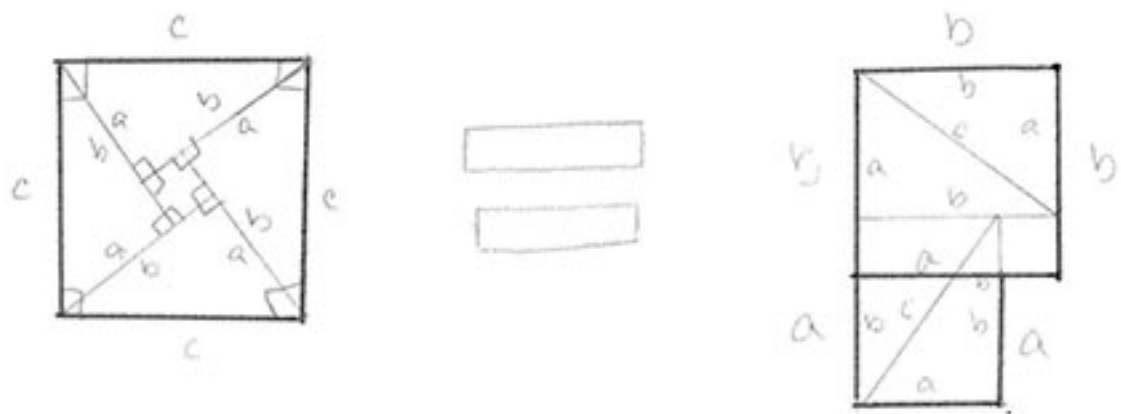
Notice there is a square inside the square, which we know is a square because the right angles of the triangles are all perpendicular to each other so it has all 90° angles. Also, we know the sides are all the same length because the length of the sides of the inside square is the difference between A and B. With the inside square, the dimensions of the square are C x C which equals C^2. Now arrange the triangles and the inside square into two squares with A and B as the sides, it should look like this:



Since the inside square is the difference between A and B, A plus the length of the inside square equals B. So in the bigger square above notice the top is side B and the other sides are side a plus the length of the inside square. In the smaller square above, the length of the bottom side is A and the length of the other sides of the smaller square are B minus the length of the inside square.

The area of the first square equals the area of the second shape because they are the same four triangles and square but in a different shape. So, the dimensions and area of the bigger square are B x B = B^2 and the dimensions and area of the smaller square are A x A = A^2. So, the area of the shape above is $A^2 + B^2$ and the area of the first square is C^2 so:

$$A2 + B2 = C2.$$



$c^2 = a^2 + b^2$