Explain the Solution to a Problem – Rubric

The **purpose** of writing explanations is twofold. First, you will notice how your own understanding grows when you try to explain your solution to someone else. You might even realize that it is not complete or has a major gap in it. Second, it will give me the means to assess your true understanding of the mathematics we are working on.

This is a core class, so it is perfectly appropriate for me to have you **writing**. I am of the opinion that many people have trouble writing because they cannot precisely express their ideas at the level of individual sentences. In mathematics there is a critical need for precision, so it offers a fertile ground for working on writing in the small. That's what we will do here.

Imagine your **audience** to be the average student in our class who has not thought about this particular problem. Mention everything necessary to make the student understand the problem and your solution. Don't assume that the professor is your audience. This will usually lead to solutions that are too short and don't show whether you truly understand the solution or not.

The main **requirements** for an explanation are:

- The problem is (re)-stated in the beginning of the explanation.
- Each sentence is true, either you explain why it is true or the sentence refers to information we used in class.
- If you are not sure if a sentence is true, you have to state that, e.g. "I believe that…" or "I conjecture that…". Be aware though that your explanation can only be complete if you are certain that all your sentences are true.
- Please show an example of your thinking to make the solution easier to understand for the reader.
- Please draw a picture if possible to make the solution easier to understand for the reader. Draw neatly and describe the pictures.
- Please show that you invested effort. If you cannot solve the problem you can for instance show all your attempts and explain what did not work.
- The writing is handwritten and legible or typed.
- Your explanation shows clear organization: In which order should the reader read the sentences and look at the pictures?
- The explanation contains few, if any spelling or grammatical errors.
- All sentences are complete (not fragments), even if you write equations and refer to pictures.
- All quantities are clearly identified; in particular, the identity of all pronouns is unambiguous. For example: "*I know this works because it is going up*". What do you mean by "this"? And what do you mean by "it"?
- The explanation ends with a sentence that wraps it all up, e.g. "I think that I solved the problem correctly because…", "This answer makes sense to me, because…"

Your explanation can also contain **other parts**:

- You can show the history of your solution process, for example "First I tried…", "But then I noticed…", "I changed my thinking because…".
- You can reference other people's work: "My partner showed me that…", "My groups had the idea…"
- Use counter examples to show that a statement is false.

Suggestions:

- Work on a first draft. Admit honestly any lack of understanding.
- Ask any questions you have before you hand in your solution. Ask first fellow students and then your instructor.
- Avoid "key words" as a substitute for an explanation. Example: "This is true because you can *cross multiply*."

Grading

Mathematical Correctness and Completeness	Have you presented a complete, legitimate mathematical solution to the problem? Did you make sure that every sentence is true? Are your arguments logical?
Depth of Understanding	Does your solution demonstrate an understanding of both the problem and your proposed solution? Are there important issues that you have neglected to consider? Does the reader believe that you truly understand the problem and its solution?
Content Presentation	Did you present pictures and examples to explain your thinking? Did you define any variables or new terms you are using? Did you check if it is clear what your pronouns refer to?
Neatness, Organization, Grammar, Spelling	Is you writing legible and organized? Did you present the problem in the beginning of your explanation? Did you check grammar and spelling? Are all your sentences complete?
Effort	Did you show that you invested effort?

In each of these categories you will be awarded between 0 and 5 points on the following scale:

- 5 - outstanding in every aspect of this category;
- 4 - very good, but there is room for improvement in this category;
- 3 - adequate, you have satisfied the requirements, but with substantial problems in this category;
- 2 - marginal, you have serious problems that have adversely effected your solutions in this category;

- 1 - unacceptable;
- 0 - essentially no work completed in this area.

Missing solutions will count negatively against your scores in all of these areas.
Notice, that within this grading scheme, approximately 60% of your grade is determined by purely mathematical issues. The other 40% is determined by issues related to your presentation of your mathematical solutions. If you do not make a conscientious effort to express your solutions in the format described above, you can receive a failing grade even if your solutions are "correct" in a mathematical sense.

Example of an Explanation

Problem:
Why do you have to find a common denominator when you add fractions?

Solution:
If you add objects, i.e. count how many there are altogether, you have to make sure that the objects are of the same kind. I often hear people say that you can't add apples and oranges. Well, technically you can: 2 apples and 4 oranges are 6 pieces of fruit, right? But you can't say that it would be 6 apples or 6 oranges. So if I want to be precise about the kind of object I am dealing with, then I need to make sure that all the objects I want to add are of the same kind.

If I, for instance, add ¼ to ½ I can think of ½ as two fourths and so I am adding one fourth to two fourths which will give me 3 fourths. In the picture below you can see how ½ (the blue area) is equal to two fourths (the blue area split up). Now we can count in fourths (which are quarter circles in the picture) and we get a total of 3 fourths.

Given any two factions, I have to change the fractions into equivalent fractions until the two fractions share the same denominator. If two fractions have the same denominator, they are referring to the same part of the whole. They might have different amounts of those parts, but since the parts are the same they are referring to the same kind of object and I can add them.