

## Realizing Mathematical Reality

Julian F. Fleron

In *A Mathematician's Lament*, Paul Lockhart has written a provocative and important critique of contemporary mathematics education in America. In his review in the April 2013 *Notices*, William Schmidt calls Lockhart's book "realistic," saying "I share in much of the author's lament," and proclaiming "the author provides an accurate characterization of mathematics instruction in the United States." Having accepted Lockhart's diagnosis - the dominant paradigm driving our system of mathematics education is irrevocably broken - the balance of Schmidt's review is, unfortunately, an exercise in avoidance.

To call Lockhart's "Mathematical Reality" too "abstract" and "unrealistic" is disingenuous. Schmidt repeatedly uses psychological projection, defensively ascribing the shortcomings of our current system onto Lockhart's vision for something new. Diversions such as these keep us from breaking out of the confines of an admittedly broken paradigm. As many examples in the history of science illustrate, if we continue blithely down this dysfunctional road, as Schmidt would seem to have us, then *we* are Alice and have gone down the rabbit-hole, not Lockhart.

Lockhart is not simply an idealist who has thrown his hands up. Throughout his lament he carefully identifies some of the important assumptions that underlie the current paradigm which we should reject *and* he identifies assumptions - which he collects under the moniker "Mathematical Reality" - that could serve as part of the foundation for a more successful paradigm.

Schmidt expects too much of one thoughtful messenger. It is the responsibility of our entire community to honestly respond to these challenges, to determine how educational practice may provide alternatives, and to help understand what new approaches could look like on a "day-to-day instructional level."

Existing programs developed under assumptions similar to Lockhart's "Mathematical Reality" can inform our efforts. One such initiative is the National Science Foundation supported project *Discovering the Art of Mathematics*. Its inquiry-based approach requires students "actually do some mathematics, and come up with their own ideas, opinions and reactions" in contexts which celebrate mathematics' "history, philosophy, thematic development, aesthetic criteria and current status." (p. 40). Curriculum materials sufficient to teach 10 semester-long courses on entirely different mathematical subject areas are freely available. Professional development workshops and other supporting resources are also available. (See <http://artofmathematics.westfield.ma.edu> .)

Projects like this vividly illustrate the potential of alternatives, like Lockhart's "Mathematical Reality," to transform what has been a long "nightmare" for students into an intellectual experience in which they feel "exultation."

Julian F. Fleron

Westfield State University

First version submitted to Notices AMS. 627 words.

Response from Steven Krantz:

Dear Julian,

I appreciate your sentiments, and would like to publish your letter. But it is too long by at least 50%. Our word limit for letters is 400 words. Please edit it down and then we can publish it.

### Realizing Mathematical Reality

In *A Mathematician's Lament*, Paul Lockhart has written a provocative and important critique of contemporary mathematics education in America. In his review of this book in the April 2013 *Notices*, William Schmidt shares “two reactions.” His first calls Lockhart’s view “realistic,” saying “I share in much of the author’s lament,” and proclaiming “the author provides an accurate characterization of mathematics instruction in the United States.” The thesis of Lockhart’s lament is that the dominant paradigm driving our system of mathematics education is irrevocably broken. Schmidt’s review makes clear that he accepts this diagnosis, as I think we should.

Unfortunately, Schmidt’s second reaction, that Lockhart’s vision of a system focused on what he calls “Mathematical Reality” as too “abstract” and “unrealistic,” is perfunctory and antithetical to the first, showing he misses the larger importance of Lockhart’s contribution. Schmidt’s use of such vague terms as well as repeated psychological projection, defensively ascribing the shortcomings of our current system onto Lockhart’s vision for something new, is unfortunate. Such diversions keep us from confronting a major shortcoming - that we are too willing to judge alternatives through the lenses of an admittedly broken paradigm. As any number of examples in the history of science illustrate, if we continue blithely down this dangerous road, as Schmidt would seem to have us, then *we* are Alice and have gone down the rabbit-hole, not Lockhart.

Instead, shouldn’t we ask: What assumptions that underlie the current paradigm should be rejected? What assumptions could serve as a foundation for a more successful paradigm? Lockhart is not simply an idealist who has thrown his hands up. Throughout his lament he carefully identifies some of the important assumptions to reject *and* he identifies assumptions - which he collects under the moniker “Mathematical Reality” - that must be part of a new system to be successful.

It is this type of visionary process, together with on-the-ground experiments to test new approaches, that are required to construct a more successful system. This is the responsibility of our whole community, not to be laid entirely at the feet of one thoughtful messenger.

And, in fact, examples which operate under alternative assumptions do exist. Within them we can begin to evaluate the legitimacy of Schmidt's concerns regarding the nature of alternatives at the "day-to-day instructional level" and whether they "would be accessible and interesting to all students."

One such initiative is Discovering the Art of Mathematics, a project whose primary audience is Mathematics for Liberal Arts students and which has been supported since 2009 by the National Science Foundation and Mr. Harry Lucas. At its center are inquiry-based materials whose investigations are the impetus for students "To actually do some mathematics, and to come up with their own ideas, opinions and reactions." (p. 40) Within this student-centered context, our subject's rich "history, philosophy, thematic development, aesthetic criteria and current status" (p. 40) is celebrated. Currently, inquiry-based materials - sufficient to teach 10 semester-long courses on entirely different mathematical subject areas - are freely available. Since many, like Schmidt, will find it hard to envision what such "Mathematical Realities" might look like or feel like, vignettes and videos of these classrooms are available online. Professional development workshops - where participants engage in this type of mathematical inquiry as both students and teachers - are being widely offered. (Information about the project, including free .pdfs of all materials, is available at <http://artofmathematics.westfield.ma.edu> .)

Results from this project are quite positive. These mathematics for liberal arts students, in what is often their last formal experience in mathematics as they join the ranks of taxpayers, parents, and voting citizens, do not experience another scene from a long nightmare. Instead, they find they really see mathematics for the first time, and they feel... exultation.

Julian F. Fleron

Westfield State University



## Realizing Mathematical Reality

In *A Mathematician's Lament*, Paul Lockhart has written a provocative and important critique of contemporary mathematics education in America. In his review of this book in the April 2013 *Notices*, William Schmidt shares “two reactions.” His first calls Lockhart’s view “realistic,” saying “I share in much of the author’s lament,” and proclaiming “the author provides an accurate characterization of mathematics instruction in the United States.” Unfortunately, Schmidt’s second reaction shows he has missed the larger purpose of the book altogether.

The thesis of Lockhart’s lament is that the dominant paradigm driving our system of mathematics education is irrevocably broken. Schmidt accepts this prognosis but then dismisses Lockhart’s vision of a system focused on “Mathematical Reality” as too “abstract” and “unrealistic.” This is psychological projection, defensively ascribing the shortcomings of our current system onto Lockhart’s vision for something new. More disturbingly, this diversion suggests Schmidt’s willingness to judge alternatives through the lenses of an admittedly broken paradigm. As any number of examples in the history of science illustrate, if we continue blithely down this dangerous road, as Schmidt would seem to have us, then *we* are Alice and have gone down the rabbit-hole, not Lockhart.

Instead, shouldn’t we ask: What assumptions that underlie the current paradigm should be rejected? What assumptions could serve as a foundation for a more successful paradigm? Lockhart is not simply an idealist who has thrown his hands up. Throughout his lament he carefully identifies some of the important assumptions to reject *and* he identifies assumptions - which he collects under the moniker “Mathematical Reality” - that must be part of a new system to be successful.

It is this type of visionary process, together with on-the-ground experiments to test new approaches, that are required to construct a more successful system. This is the responsibility of our whole community, not to be laid entirely at the feet of one thoughtful messenger.

And, in fact, examples which operate under alternative assumptions do exist. Within them we can begin to evaluate the legitimacy of Schmidt’s concerns regarding the nature of alternatives at the “day-to-day instructional level” and whether they “would be accessible and interesting to all students.”

One such initiative is *Discovering the Art of Mathematics*, a project whose primary audience is Mathematics for Liberal Arts students and which has been supported since 2009 by the National Science Foundation and Mr. Harry Lucas. At its center are inquiry-based materials whose investigations are the impetus for students “To actually do some mathematics, and to come up with their own ideas, opinions and reactions.” (p. 40) Within this student-centered context, our subject’s rich “history, philosophy, thematic development, aesthetic criteria and current status” (p. 40) is celebrated. Currently, inquiry-based materials - sufficient to teach 10 semester-long courses on entirely different mathematical subject areas - are freely available.

Since many, like Schmidt, will find it hard to envision what such “Mathematical Realities” might look like or feel like, vignettes and videos of these classrooms are available online. Professional development workshops - where participants engage in this type of mathematical inquiry as both students and teachers - are being widely offered. (Information about the project, including free .pdfs of all materials, is available at <http://artofmathematics.westfield.ma.edu> .)

Results from this project are quite positive. These mathematics for liberal arts students, in what is often their last formal experience in mathematics as they join the ranks of taxpayers, parents, and voting citizens, do not experience another scene from a long nightmare. Instead, they find they really see mathematics for the first time, and they feel... exultation.

Julian F. Fleron

Westfield State University

### Realizing Mathematical Reality

In *A Mathematician's Lament*, Paul Lockhart has written a provocative and important critique of contemporary mathematics education in America. In his review of this book in the April 2013 *Notices*, William Schmidt shares “two reactions.” His first calls Lockhart’s view “realistic,” saying “I share in much of the author’s lament,” and proclaiming “the author provides an accurate characterization of mathematics instruction in the United States.” Unfortunately, Schmidt’s second reaction shows he has missed the larger purpose of the book altogether.

The basis of Lockhart’s lament is that the dominant paradigm driving our system of mathematics education is irrevocably broken. Schmidt accepts this prognosis but then dismisses Lockhart’s vision of a system focused on “Mathematical Reality” as too “abstract” and “unrealistic.” This is psychological projection, defensively ascribing the shortcomings of our current system onto Lockhart’s vision for something new. Throughout *A Mathematician's Lament* Lockhart uses thoughtful Socratic-like dialogues to anticipate the expected questions and retorts from critics. As Socrates is to Meno, Lockhart draws out what we already know. To continue blithely along, heedless to this dominant, but admittedly broken system, shows that we are Alice and have gone down the rabbit-hole, not Lockhart.

Instead, shouldn't we ask: What assumptions upon which the current paradigm is built should be rejected? Upon what new assumptions might a more successful paradigm be constructed? Lockhart is not simply an idealist who has thrown his hands up. Throughout his lament he carefully identifies some of the important assumptions to reject *and* he identifies assumptions - which he collects under the moniker “Mathematical Reality” - that must be part of a new system to be successful.

It is this visionary process, together with on-the-ground experiments to test new approaches, that are required to construct a more successful system. This is the responsibility of our whole community, not to be laid at the feet of one thoughtful messenger.

And, in fact, examples which operate under alternative assumptions do exist. Within them we can begin to evaluate the legitimacy of Schmidt's concerns regarding the nature of alternatives at the "day-to-day instructional level" and whether they "would be accessible and interesting to all students."

One such initiative is Discovering the Art of Mathematics, a project whose primary audience is Mathematics for Liberal Arts students and which has been supported since 2009 by the National Science Foundation and Mr. Harry Lucas. At its center are inquiry-based materials whose investigations are the impetus for students "To actually do some mathematics, and to come up with their own ideas, opinions and reactions." (p. 40) Within this student-centered context, our subject's rich "history, philosophy, thematic development, aesthetic criteria and current status" (p. 40) is celebrated. Currently, inquiry-based materials sufficient to teach 10 semester-long courses on entirely different mathematical subject areas are freely available. Since many, like Schmidt, will find it hard to envision what such "Mathematical Realities" might look like or feel like, vignettes and videos of these classrooms are available online. Professional development workshops - where participants engage in this type of mathematical inquiry as both students and teachers - are being widely offered. (Information about the project, including free .pdfs of all materials, is available at <http://artofmathematics.westfield.ma.edu> .)

Results from this project are quite positive. Mathematics for liberal arts students, in what is often their last formal experience in mathematics as they join the ranks of taxpayers, parents, and voting citizens, do not experience another chapter in a long nightmare. Instead, they find they really see mathematics for the first time, and they feel... exultation.

Julian F. Fleron

Westfield State University

We hope that

The project described here is not a panacea, it is audience dependent and is not readily transferable. But it illustrates that successful “Mathematical Realities” can and do exist. Unless we foster more opportunities to build and explore them, we will be unable to see what is available beyond the current, broken paradigm.

If you accept Lockhart’s arguments, as Schmidt apparently does, it is true, it means we are operating within a broken paradigm.

Schmidt worries that Lockhart’s “Mathematical Reality” will exacerbate “inequalities in mathematics education that are currently rampant” and that mathematics would “desert many others?” Again, Lockhart anticipated such absurdities in support of what is broken. He says, “If I had to design a mechanism for the express purpose of *destroying* a child’s natural curiosity and love of pattern-making, I couldn’t possibly do as good a job as is currently being done” (p. 20; author’s emphasis) and “They’re not learning anything now! ...The mathematics has already been removed!” (p. 35)

In his review, Smith has many positive things to say about Lockhart’s provocative *A Mathematician’s Lament*. [Required reading?]

Smith critiques come in the form of two shortcomings. First, that Lockhart does not “provide a vision for what can actually be done.” This is much too much to expect from any one person and should not be considered a shortcoming. Rather, I would argue that Lockhart should be lauded for the courage to challenge the absurdity of a status quo that is so dysfunctional for most of society. Morris Kline’s worry that we \_\_\_\_ have now been more than fully realized. Lockhart throws down a gauntlet for change.

Second, Smith cannot envision “mathematical playground.” That we have gone so far down this path of ruin the fact that we cannot even envision a different mathematical world is a debilitating position that precludes the potential for change. But such mathematical playgrounds do exist. The growth in Math Circles is one example. The NSF funded *Discovering the Art of Mathematics* project helps nurture such playgrounds for collegiate mathematics for liberal arts students. At our home institutions our mathematics for liberal arts students read *A Mathematician’s Lament* as required reading. They are largely furious to have had the mathematical educations they have, which has generally driven them away from STEM fields,

when the world described by Lockhart, and offered by our course, should have been available to them. With the opportunity to play in this world, at least for a semester, they leave transformed – citizens, future parents, and taxpayers who have a fundamentally different view of mathematics. [Our undeclared liberal arts students in the course to fulfill core requirements is becoming one of our most fertile grounds for recruiting strong mathematics majors.]

As a community we have much to learn from these alternative mathematical worlds.

URL to our project?

To surveys?

Send this to David Farmer and whatshername.

Mathematicians are unrealistic when the world of mathematics that constitutes their profession is so disjoint from the world of mathematics that they train their students in.

Purpose of Writing a Response: Set the record straight on the big picture that Schmidt missed to set the record straight AND THEN use this as an opportunity to publicize DAoM.

495 Words

In his review of A Mathematician's Lament, William Schmidt seems moved by Paul Lockhart's provocative and important critique of contemporary mathematics education in America. Schmidt calls the lament "realistic," says "I share in much of the author's lament," and proclaims "the author provides an accurate characterization of mathematics instruction in the United States." Unfortunately, his view that Lockhart is also "unrealistic" shows that while he sees the critique, he has missed the larger purpose of the book.

Schmidt complains that Lockhart's "Mathematical Reality" is abstract. Mathematical reality too abstract? Unrealistic? Don't you see Prof. Schmidt, it is a metaphor. Lockhart is mocking us. We are so subsumed by the dominant, broken paradigm that we don't realize that we are Alice and have gone down the rabbit-hole, not Lockhart.

Schmidt worries that Lockhart's "Mathematical Reality" will exacerbate "inequalities in mathematics education that are currently rampant" and that mathematics would "desert many others?" Again, Lockhart anticipated such absurdities in support of what is broken. He says, "If I had to design a mechanism for the express purpose of *destroying* a child's natural curiosity and love of pattern-making, I couldn't possibly do as good a job as is currently being done" (p. 20; author's emphasis) and "They're not learning anything now! ... The mathematics has already been removed!" (p. 35)

If we can admit to the magnitude of the problem and quit clinging to what is irrevocably broken, then Schmidt's questions about what "Mathematical Reality" would look like "at the day-to-day instructional level", and whether it "would be accessible and interesting to all students" become relevant.

One large-scale effort that can serve as an example is Discovering the Art of Mathematics which has been supported since 2009 the National Science Foundation and Mr. Harry Lucas. This project nurtures collegiate Mathematics for Liberal Arts courses and classrooms which are very much in the spirit of Lockhart's "Mathematical Reality". Currently, inquiry-based learning materials sufficient to teach 10 semester-long courses on entirely different mathematical topics are freely available. Professional development workshops are being widely offered. Classroom videos which illustrate what such classrooms are like are forthcoming. (All information about the project, including .pdfs of the materials, are available at <http://artofmathematics.westfield.ma.edu> .)

At Westfield State, the project's home institution, the results are significant. Students report significant affective gains, significant confidence gains, and new appreciation for the role of the aesthetic in mathematics. Many feel exultation, to borrow a term from Lockhart. Failure and withdraw rates are significantly different than in control groups. Soon to be taxpayers and parents, recently enrolled in the voting ranks, these students leave - what for most will be their final formal experience in mathematics - fundamentally changed.

The project described here is not a panacea, it is audience dependent and is not readily transferable. But it illustrates that successful "Mathematical Realities" can and do exist. Unless we foster more opportunities to build and explore them, we will be unable to see what is available beyond the current, broken paradigm.

Having, admittedly, demolished the dominant paradigm propped up by a massive establishment, it's disingenuous to level your critic's philosophical alternatives as "unrealistic" by laying the massive failures of your paradigm at his feet. Instead, confronted with the

evidence mature scientists would try to look with new eyes, seek new models, and try new experiments.

Dad and Kim

Last sentence can be really damaging. Need to explain audience dependent and not transferrable is. Break it down into sentences. They had no idea what it was saying.

Schmidt is making an all or nothing argument. Trying to analyze too much. Stuck in the old structure.

Reading Lockhart showed Kim why I do what I do.

Unworkable and irrevocably broken.

Laying it at the feet of Lockhart is good.

Wholeheartedly accepts Lockhart's demollision of the current paradigm.

Rejecting assumptions on which the original paradigm was build. So what do we reject?

Without rejections of assumptions... You have to outright reject the assumptions. If you mess around too much, you perpetuate the frog's eye view. You need to see the larger context.

Metaphors carry a whole lot of stuff with them. "Informing tropes". Sourcier – a Swiss linguist.

We are so hung up in the details that we can't get out of our way.

Not being able to realize the absurdity that you are stuck in. Unreality. What is the word for this? Transference – unconscious redirection of feelings from one person to another. Psychological projection – defense mechanism which (Freud) where a person subconsciously denies his or her own negative attributes by ascribing them to others. Project faults onto others.

Theatre of the Absurd.

Where is Schmidt's vision? Leadership?

How dare you? Prime facia absurd.

What are you going to do about it? It's easy to talk about it being broken. Throwing up your hands. We won't do anything since you can come up with a better idea. Lockhart has CLEARLY isolated some of the assumptions that should be rejected for us to move forward!! (List some of these critical things.)

Recognizing and rejecting assumptions. That is what DAoM has done.

Foundations of the problem.

It's writing a song. Writing a poem. This distills my entire philosophy of teaching. All I know about philosophy of science. And gives the entire rationale for the project in the first place.

Our project plays a critical role as we try to have a larger discussion about how to address the glaring weaknesses pointed out by Lockhart.

This is the rationale for why we are working so hard on this project. Under different assumptions, what is really possible?

Kuhnian Terms

Normal Science

Crisis

Revolution

## Egalitarianism of Mathematics

In the February 2013 issue of the *Notices*, it is claimed in an Opinion piece by the editor and in a Letter to the Editor that mathematics as a subject is egalitarian. It may appear so to those inside the wall as they greet each other within the cloister's walls but it certainly does not appear to be so to those of us outside.

The editor assumes that mathematics-as-subject is independent of mathematics-as-profession. He then argues that while the profession is not egalitarian, because of this independence, the subject can be and, in his view, is. The author of the Letter to the Editor says mathematics is egalitarian because it is "open to all with the requisite talent and training." For this writer, the work itself—the talent—does not suffice. It seems to me that the requirement of training asserted by the second author denies the independence of subject and profession asserted by the first. Which one is closer to the truth?

There is no empirical foundation to mathematics against which contributions can be measured in an objective manner. This is not true of other sciences. If one discovers a new comet or a new bird species one is acknowledged as having contributed to astronomy or ornithology and is considered thereby to be a member of the community. It would be absurd to assert that the discovery of a new comet is void because the person making the discovery is unable to present credentials of their training in astronomy. The empirical sciences can successfully argue that subject and profession are independent and that it is therefore possible for the subject to be egalitarian even if the profession is not.

Without a subject-based means to measure contribution, mathematics takes training as a proxy. It is the requirement of a credential in order to participate that binds subject to profession. Because of this binding, the nonegalitarianism of the latter comes to cloud the former. Thus I

think that the letter writer is closer to the truth than the editor.

The credentialed mathematics profession has built a wall around American mathematics as a subject. The

profession is the gate-keeper of the journals, the grants, the conferences, and even online repositories such as arXiv. Mathematics as a subject is not egalitarian because how it is defined and what it contains is determined solely by a nonegalitarian profession.