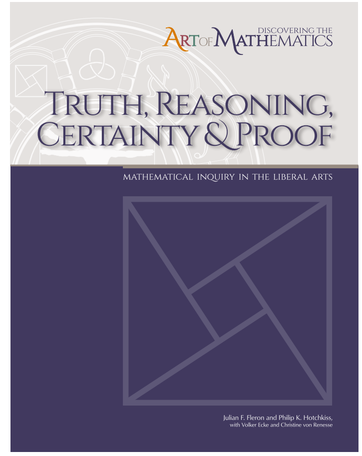
The following is excerpted from **Discovering the Art of Mathematics: Student Toolbox** which is forthcoming. Similar activities can found in:



Discovering the Art of Mathematics: Truth Reasoning, Certainty & Proof

By Julian F. Fleron and Philip K. Hotchkiss, with Volker Ecke and Christine von Renesse

As with all of our learning guides, this book is freely available online at

`http://www.artofmathematics.org/books/`

Discovering the Art of Mathematics (DAoM) is an NSF supported project that supports inquiry-based learning (IBL) approaches for mathematics for liberal arts (MLA) courses.

The DAoM curriculum consists of a library of 11 inquiry-based learning guides. Each volume is built around deep mathematical topics and provides materials which can be used as content for a semester-long, themed course. These materials replace the typical lecture dynamic by being built on inquiry-based investigations, tasks, experiments, constructions, data collection and discussions.

DAoM also provides a wealth of resources for mathematics faculty to help transform their courses. Extensive online resources include volume specific teacher notes and sample solutions, classroom videos of IBL in action, sample student work, regular blogs about teaching using IBL and a regular newsletter. Opportunities for supported reviewing and beta testing are also available.

For departments interested in IBL, DAoM offers traveling professional development workshops.

Full information about the *Discovering the Art of Mathematics* project is available at

`http://www.artofmathematics.org`

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Use the excerpt below to explore for yourself how our materials can engage students in mathematical inquiry.

Pennies and Paperclips

Pennies and Paperclips* is a two player game played on a board resembling a checkerboard. One player, "Penny", gets two pennies as her pieces. The other player, "Clip", gets a pile of paperclips as his pieces. Penny places her two pennies on any two different squares on the board. Once the pennies are placed, Clip attempts to cover the remainder of the board with paperclips - with each paperclip being required to cover two adjacent squares. Paperclips are not allowed to overlap. If the remainder of the board can be covered with paperclips then Clip is declared the winner. If the remainder of the board cannot be covered with paperclips then Penny is the winner.

1. With a partner, play this game several different times. Record the results of your games, including the placement of the pennies a paperclips, on the miniature boards in Figure 2.

2. Now switch the roles of pennies and paperclips and play several more games, again recording your results.

3. Do you notice any patterns that will enable you to find winning strategies for the players? If so, test them by playing a few more games. If not, continue to play until some pattern appears.

4. State a conjecture which determines precisely when pennies win based on their placement.

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*The history of this game is not known to the author. It appears in the 1994 version of Harold R. Jacobs' Mathematics: A Human Endeavor. In the 1970 version of this book the "mutilated checkerboard" appears instead. The latter is from the 1950's and its notoriety is due to Martin Gardner's *Scientific American* columns. It may well be that the translation of the checkerboard problem into a game is due to Jacobs.

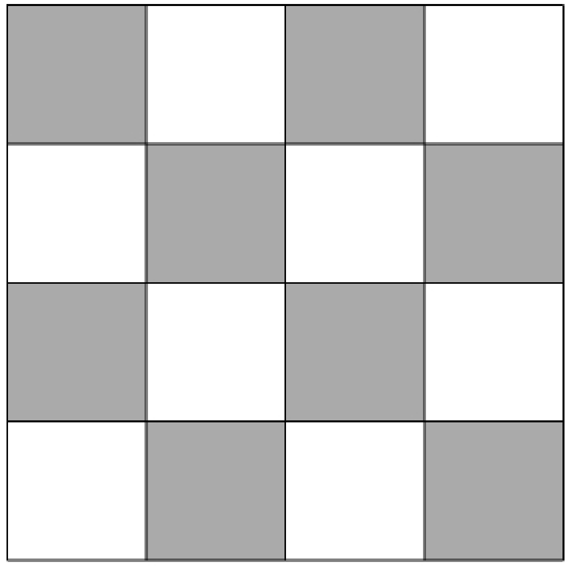

Figure 1: Beginner board for Pennies and Paperclips game.

5. Prove this conjecture.
6. State a conjecture which determines precisely when paperclips win based on the placement of the pennies.
7. Prove this conjecture.
8. Are there any situations in which neither player wins, or have you characterized all possible outcomes? Explain.

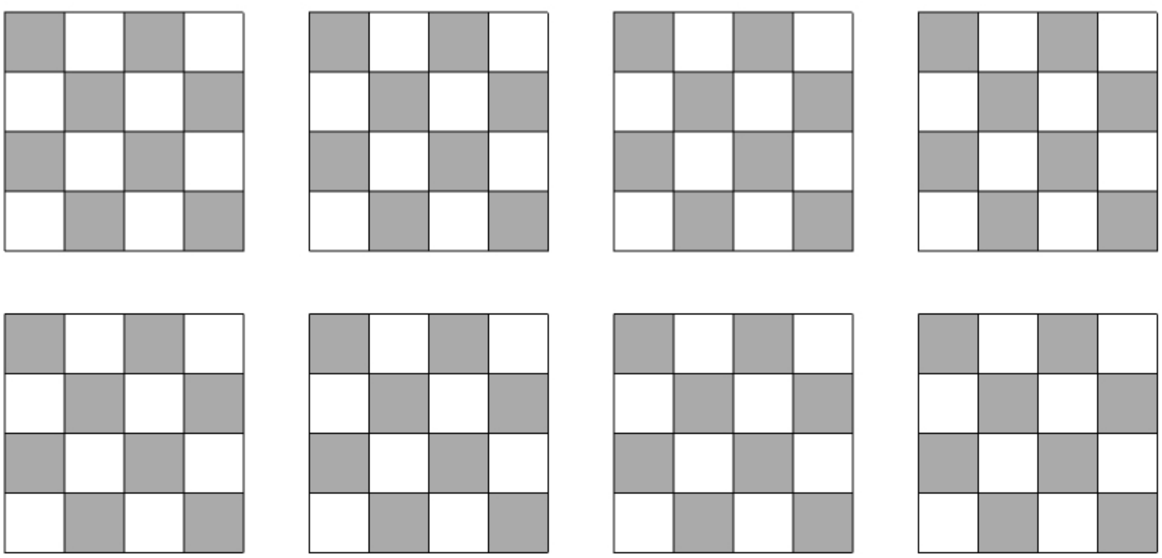

Figure 2: Board for recording Pennies and Paperclips games.

It is important to remember that a proof is a logically complete demonstration that a result must hold. Proof by example is not allowed. The proof most people originally give for Investigation **7** is merely a counting argument.

9. Play pennies and paperclips a few times on the distorted board in Figure 3.

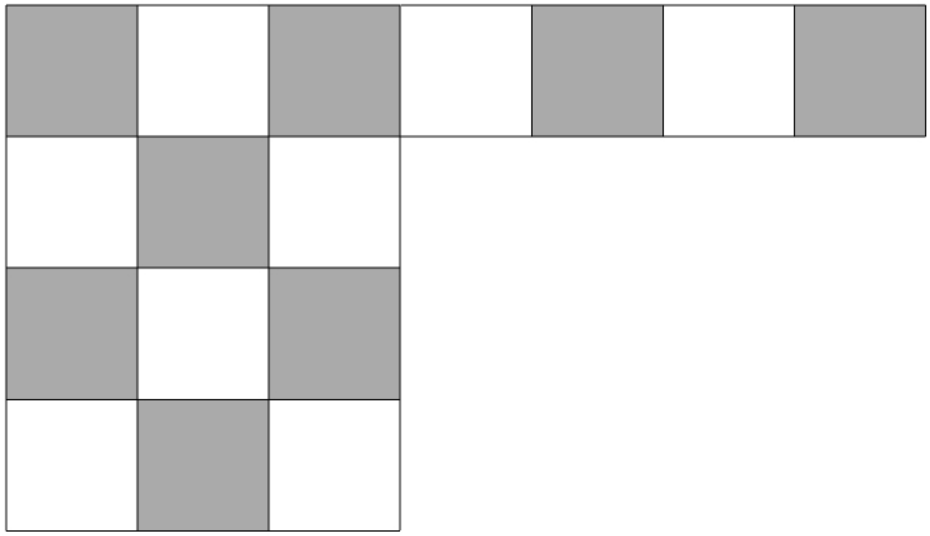

Figure 3: Distorted beginner board for Pennies and Paperclips game.

10. Does your conjecture in Investigation **6** hold for this board?

11. Return to your proof in Investigation **7**. Does your proof fundamentally use the geometry of the board or is it simply a counting argument? The distorted board generally shows that the "proofs" in Investigation **7** are not complete.

12. If your proof in Investigation **7** is not complete, see if you can use the *Hamiltonian circuit* to show that paperclips can be appropriately placed to win the expected games.

The board you have been playing on was called the "beginner" board. Mathematicians love to generalize. It is a very natural mathematical question to ask whether this game can be extended to other sized boards. Try other sized boards and see what you find!