# Discovering the Art of Mathematics

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**http://artofmathematics.wsc.ma.edu/**

Funded by a National Science Foundation curriculum development grant, the *"Discovering the Art of Mathematics"* project[1] is developing a library of ten inquiry-based learning guides whose primary use is intended to be with collegiate Mathematics for Liberal Arts students. The following six learning guides are now ready for beta-testing:

| | | |
|---|---|---|
| Number Theory, | The Infinite, | Knot Theory, |
| Games and Puzzles, | Music and Dance, | and Geometry. |

PDF copies are available for review from our web site at
http://artofmathematics.wsc.ma.edu/

Please contact us if you have questions about any of these materials, or if you are interested in beta-testing portions of these learning guides in your classes. We have some money in our grant to support your beta-testing activities. In addition, early draft materials for learning guides on *Art and Sculpture, Patterns, Reasoning,* and *Calculus* on our web site, as well.

**Thursday January 6, 2011, 2:15 p.m.-6:10 p.m.**
MAA Session on The Mathematics of Games and Puzzles, I
Grand Chenier Room, 5th Floor, Sheraton
3:15 p.m. *Discovering the Art of Mathematics: Straight-Cut Origami.*
**Christine von Renesse***, **Volker Ecke**, Westfield State College

**Friday January 7, 2011, 2:00 p.m.-4:00 p.m.**
MAA Poster Session on Projects Supported by the NSF Division of Undergraduate Education
Napoleon A1-A3, 3rd Floor, Sheraton
Discovering the Art of Mathematics. **Julian F. Fleron***, **Philip K. Hotchkiss**, **Volker Ecke**, **Christine von Renesse**, Westfield State College

**Sunday January 9, 2011, 1:00 p.m.-6:00 p.m.**
MAA Session on Humanistic Mathematics, II Mardi Gras GH, 3rd Floor, Marriott
2:00 p.m. *Student Inquiry into the Limits of Knowledge - Removing Barriers in Mathematics for Liberal Arts*.
**Philip K Hotchkiss***, **Julian F Fleron**, **Volker Ecke**, **Christine von Renessee**, Westfield State College

**Sign-up:** If you would like to be kept up-to-date on the release of new materials, please add your name and email address to the sign-up sheet. We'll be happy to send occasional information about significant updates.

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[1] These materials are also based on work supported by Project PRIME which was made possible by a generous gift from Mr. Harry Lucas.

## 3. Irregular Shapes

We noticed that lines of symmetry are very helpful in folding polygons so that we can cut them out with a single straight cut. Yet, with shapes that are less symmetric, it is no longer clear how to proceed.

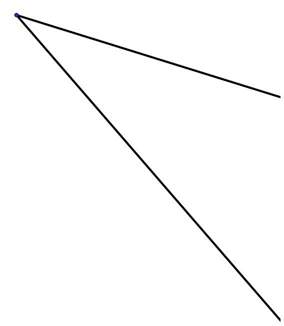


Figure 5. Single Angle for Straight-cut Origami (large: Figure 19).

**19.** Find a way to fold the single angle shape in Figure 5. How does the folding of this shape relate to the work you did with the symmetric shapes in Section 2? Explain.

You may find the following series of investigations more challenging than previous ones. It may take you more than just one attempt with each shape, sometimes *many* more. You may observe that using ideas gained from working with symmetric shapes may not be enough to fold these irregular shapes. Do not be discouraged. Where could you go for some new ideas? Do not discard the results of your attempts. Instead, use them as resources to analyze carefully what the results look like, and why they do not *completely* accomplish the task. Also keep an eye on what happens to interior and exterior areas when folding.

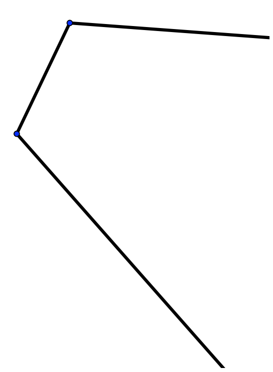


Figure 6. Double Angle for Straight-cut Origami (large: Figure 20).

Mathematicians have a name for the line (or line segment) that divides an angle into two equal parts: they call this an **angle bisector**.

**20.** Find a way to fold the double-angle shape in Figure 6.
**21.** Find a way to fold the ***irregular triangle*** in Figure 7 (three sides all of different lengths).

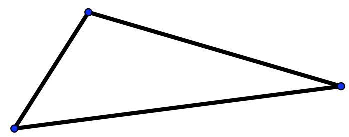


Figure 7. An Irregular Triangle for Straight-cut Origami (large: Figure 21).

**22. Classroom Discussion:** Other than angle bisectors, what other kinds of lines did you use to fold up the shapes in Figure 6 and Figure 7? How would you describe those?

**23.** An angle bisector is a line of symmetry for one angle. Is it also a line of symmetry of the *entire* shape? What are the consequences of folding along an angle bisector all the way through the shape?

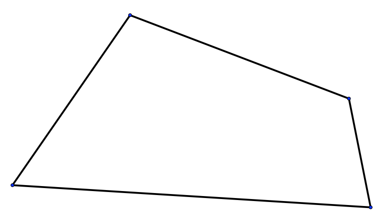


Figure 8. An Irregular Quadrilateral for Straight-cut Origami (large: Figure 22).

**24.** Find a way to fold the ***irregular quadrilateral*** in Figure 8 (four sides all of different lengths).

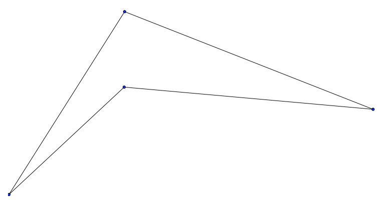
**25.** Did your observations about folding the double-angle, or the irregular triangle, help you in folding the irregular quadrilateral? Explain.

**26.** Were there any new ideas that you used for this shape? Explain.

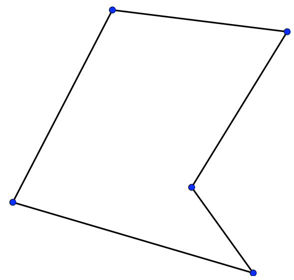
**27. Classroom Discussion:** Given a line $\ell$ and a point $P$ not on $\ell$, we can construct the **perpendicular**. This is a line through $P$ which makes a $90^\circ$ angle with $\ell$. Look back at your folding patterns and find perpendiculars. What do you notice?

**28.** Next consider the shapes in Figure 9. In what ways are these shapes different from ones you have considered in this section so far? Explain.

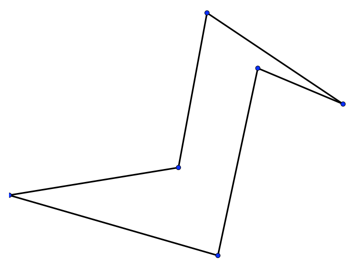
**29.** Create three geometric shape of your own that belong to this new type. Explain why each belongs to this new type.



(a) Quadrilateral (large: Figure 23).



(b) Pentagon (large: Figure 24).



(c) Hexagon (large: Figure 25).

FIGURE 9. Three Shapes for Straight-Cut Origami.

**30.** Find a way to fold the quadrilateral in Figure 9(a) so that you could cut it out with a single straight cut.

**31.** Find a way to fold the pentagon in Figure 9(b) so that you could cut it out with a single straight cut.

**32.** Find a way to fold the ***non-convex hexagon*** in Figure 9(c) (six sides, segments between some of the vertices fall outside the shape) so that you could cut it out with a single straight cut.

**33.** Once you know how to fold and cut these shapes, make another folded copy of each but *do not cut it out*. Carefully unfold it, making sure to note which of the fold lines were used in your final version, and which were not. Clearly mark all the fold lines that were needed for your final version.

**34.** **Writing Assignment:** Using the marked and labeled shapes you created in Investigation **33** as resources, clearly describe a geometric way in which these folds relate to the original lines of the polygon. Write a complete and careful summary of your observations and findings.

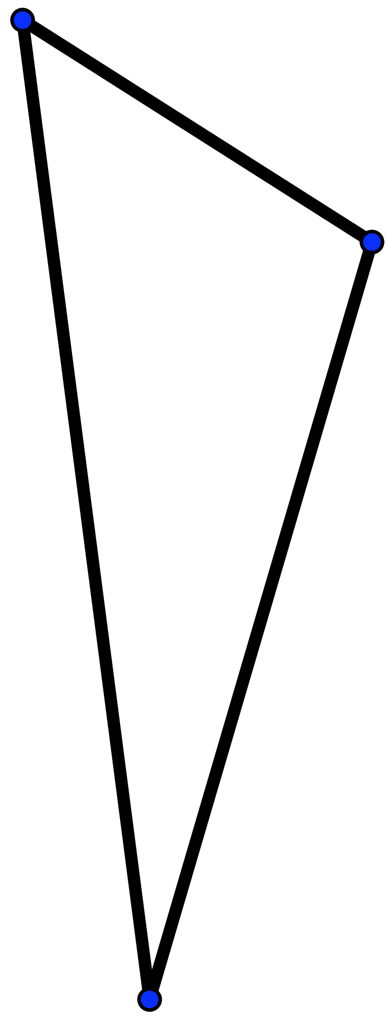


FIGURE 21. An Irregular Triangle for Straight-cut Origami.

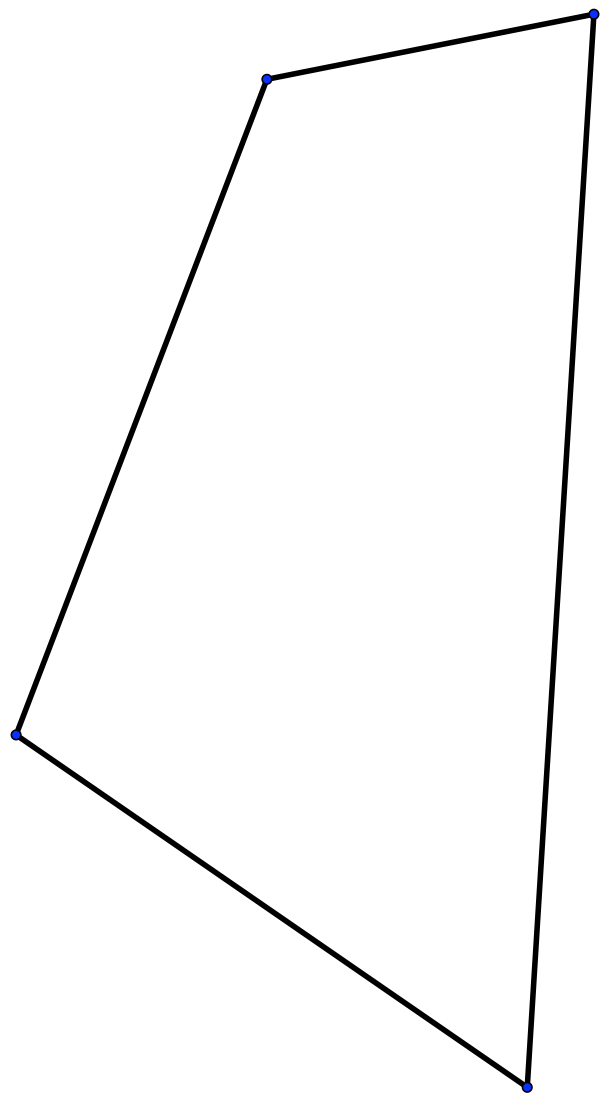


Figure 22. An Irregular Quadrilateral for Straight-cut Origami.

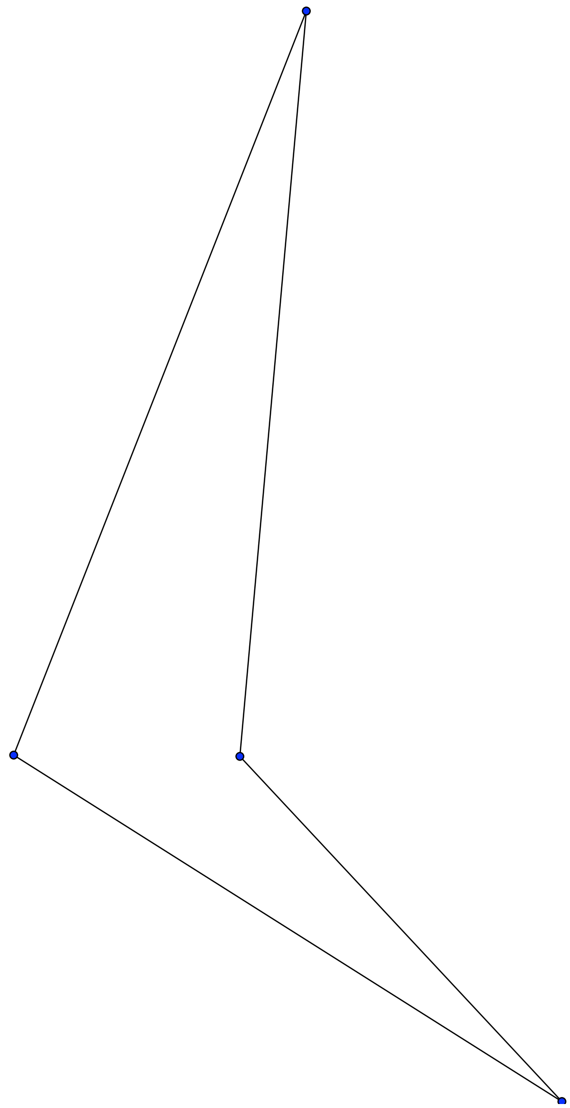


Figure 23. An Irregular Quadrilateral for Straight-cut Origami.

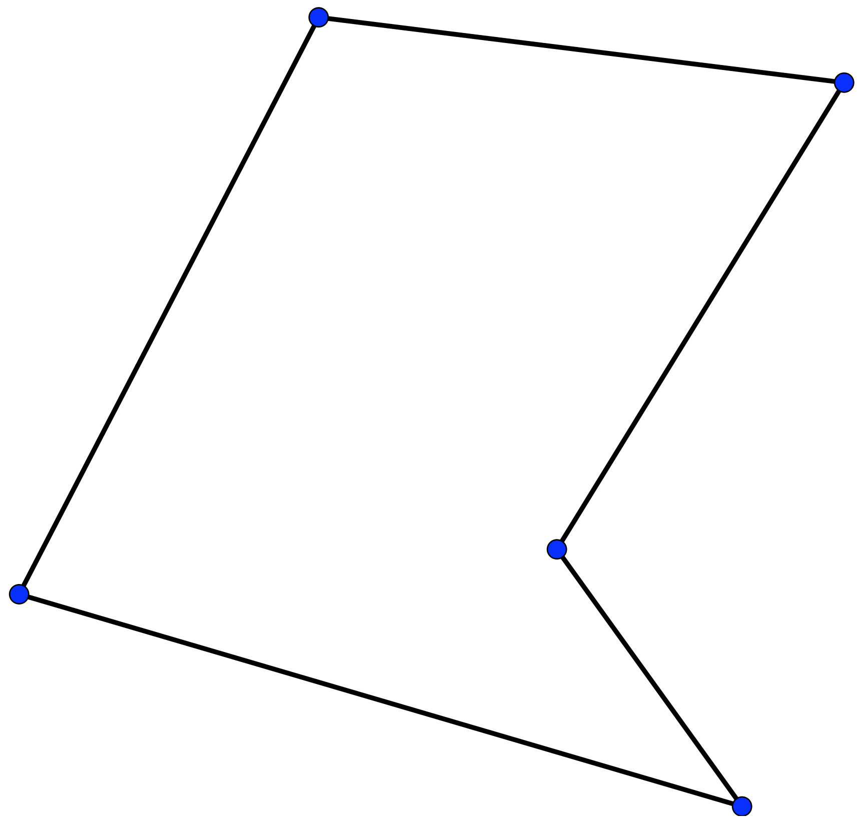


Figure 24. An Irregular Pentagon for Straight-cut Origami.

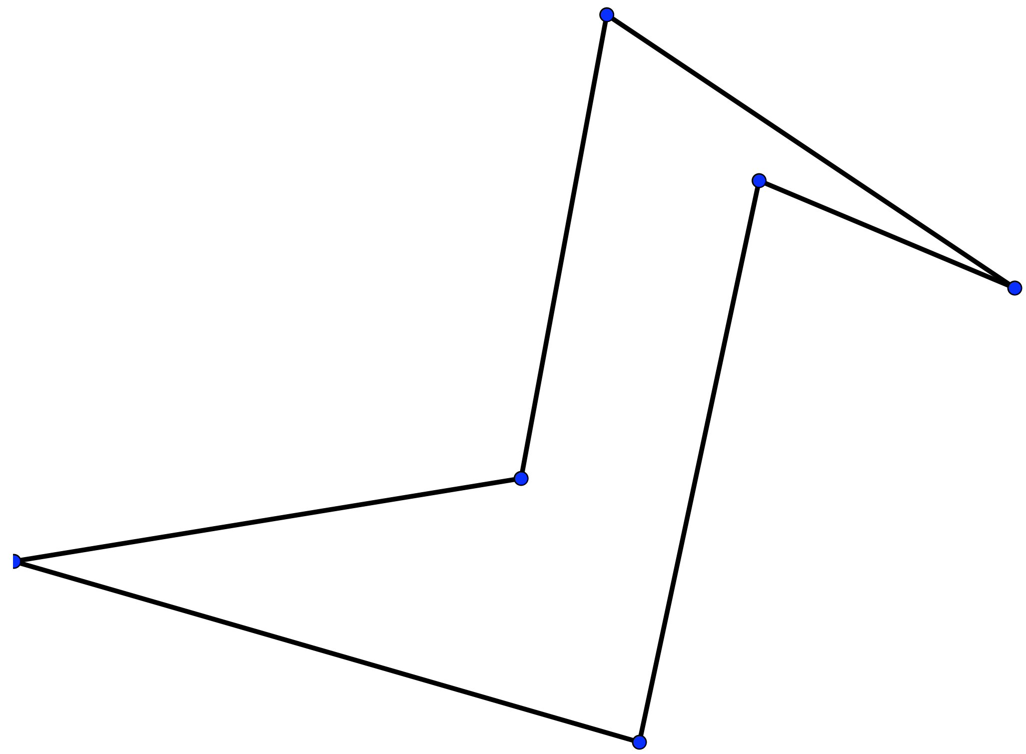


Figure 25. An Irregular Hexagon for Straight-cut Origami.

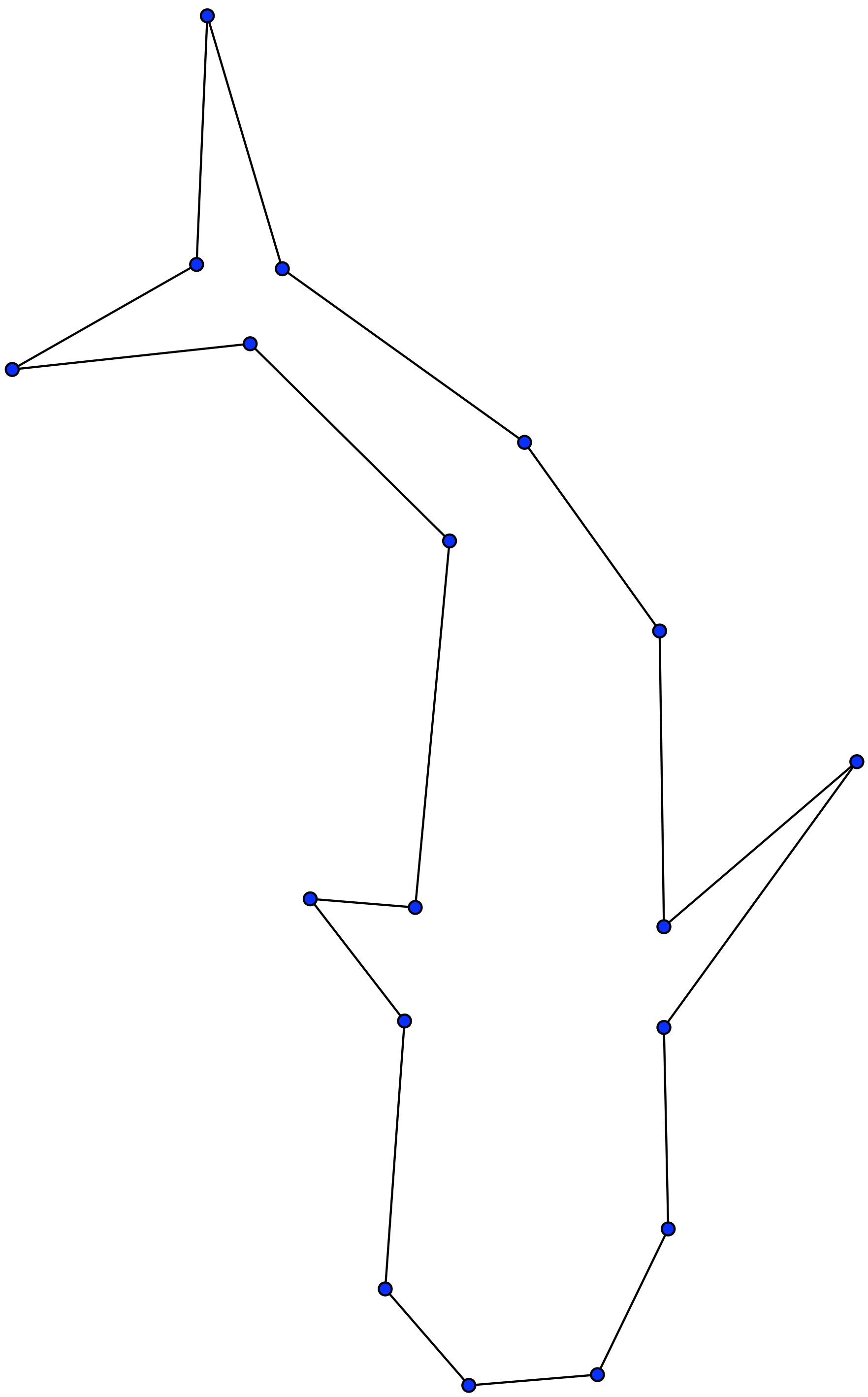


FIGURE 26. Whale for Straight-cut Origami.