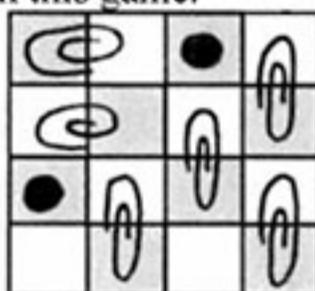


5/5

The Pennies and Paperclips Game

Introduction: The Pennies and Paperclips game is a square board resembling a checkerboard. In our game we had a 4X4 grid with 16 squares, 8 white and 8 greys. The two game pieces are Pennies and Paperclips. To start, two pennies get placed on the board in different squares and the pennies win if there are spaces that are unable to be covered by paperclips, and paperclips win if all the spaces are covered. The paperclips must be either horizontal or vertical; they cannot be placed diagonal. An example of a game where pennies wins is shown in the game below. The pennies were placed on 2 grey squares. This means that there are 6 open grey squares and 8 open white squares. As you can see, the paperclips are each covering one square of gray and one square of white and there is no way to cover the last 2 white squares while still following the rules, which means that the pennies win this game.

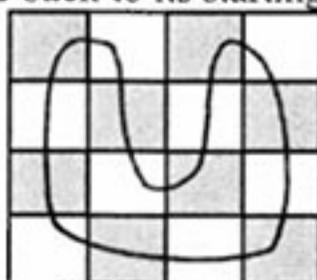


Conjecture: In the Pennies and Paperclips game, if you place the pennies in the same colored boxes then the pennies will win.

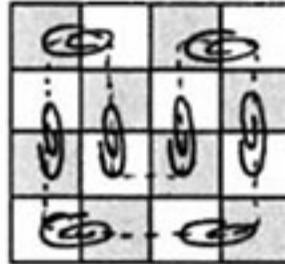
Proof: In the Pennies and Paperclips game every paper clip has to go horizontally or vertically and cannot overlap or go diagonally which means that the paper clips will always cover one grey box and one white box. When the pennies cover 2 of the same color boxes it makes the ratio of grey to white boxes uneven so that there are 8 of one color and 6 of the other. The fact that the paper clips have to be covering one white and one grey box means that having 2 extra boxes of one color makes them impossible to cover with the paper clips and means that pennies will win.

Conjecture: In our Pennies and Paperclips games, if you place the pennies on opposite color boxes, then the paperclips will win.

Proof: We are using a square game board with an even number of boxes, so we can use a winning strategy based on the Hamiltonian pathway. If the game board had an odd number of boxes this strategy would not work. The Hamiltonian pathway is a path that touches every single point, never touching a single point twice and always getting back to its starting point. For our game board we imagined that each of the individual boxes was a point and found a path that touched each point only once and came back to its starting point. Our pathway looks like this:



By using this pathway you can win every single game where the pennies are placed on opposite colors. If you follow this pathway with only the paper clips, the entire board will be filled with no overlapping paper clips or no left over boxes. As seen in the picture below:



When the pennies are placed on the opposite color boxes you lay the paper clips down along the pathway just jumping over the space with a penny and continuing along the path. When you place the pennies on opposite color boxes you are splitting the Hamiltonian pathway into two sections and each side of the path will have an even number of boxes. Having an even number of boxes in between each of the two pennies means that when you place your paper clips along the path there will always be an even number of spaces to cover and the paper clips will never have to overlap or leave empty boxes. For example in the game below the pennies were placed on one grey box and one white box. You can see that there is an even number of boxes for each section of the Hamiltonian pathway, and that each section fits the paper clips perfectly. Each time the pennies are placed on opposite colors the Hamiltonian pathway will always be cut into two even sections, and each time the paperclips will fit perfectly. The paperclips fitting perfectly means that every time the pennies are placed on opposite color squares, the paperclips will cover the board completely and they will win.

