

AFICE HUTT

3a+5b conjecture: Beginning with 8, any whole number may be a solution to 3a+5b (given that a,b > 0 or =0), in addition to the numbers 0, 3, 5, and 6. The numbers 1, 2, 4, and 7 cannot be solutions to 3a+5b when a and b are greater than or equal to zero.

0, 3, 5, and 6 can be found be setting both a and b equal to zero, setting a equal to 1 and b equal to zero, setting a equal to zero and b equal to 1, and setting a equal to 2 and b equal to zero, respectively. 1, 2, and 4 cannot be solutions because they are less than either 3 or 5, while 7 cannot be a solution because 3 and 5 are both less than 7, but combined are greater than 7.

When 3a+5b=8, a=1 and b=1. From here you can use 3a+5b to increase the solution by 1 using the following algorithm:

- Increase a by 2 (add two 3s), decrease b by 1 (subtract one 5)
- Decrease a by 3 (subtract three 3s), increase b by 2 (add two 5s)
- Increase a by 2, decrease b by 1
- Repeat

Using the above pattern, the solution to 3a+5b will increase by 1 with each successive step completed, allowing for infinite repetition to find every whole number greater than 8 as a solution. By adding two 3s and subtracting one 5, you are adding 6 and subtracting 5, so in the end 1 is the net increase (likewise with subtracting three 3s and adding two 5s, you are subtracting 9 while adding 10, so the increase is still 1). This algorithm of adding and subtracting multiples of 3 and 5 will always work because by starting at 8, a=1 and b=1. Over one cycle of the algorithm, a will increase by a total of 1, while b nets no gain:

	Α	В
8	1	1
	+2	-1
9	3	0
	-3	+2
10	0	2
	+2	-1
11	2	1

Since the lowest a reaches in the first cycle is 0, and it increases by 1 each cycle, it will never dip below 0. Since b nets no increase each time, it will cycle through 1, 0, and 2.